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On a latin square problem of Fuchs

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In memory of Anne Penfold Street

Abstract

Fuchs asked: For which integer partitions $g_1 + \cdots g_n = N$ does there exist a latin square of side N having n subsquares of sides $g_1, \ldots, g_n$ having no rows, columns, or symbols in common? Only when at most two distinct integer parts are used is the answer known completely; in general even the necessary conditions are elusive. Two conjectures giving plausible sufficient conditions are advanced. The first asserts than whenever the largest three integer parts are the same, such a latin square always exists. The second asserts that whenever the largest part is no larger than $n-2$ times the smallest, such a latin square always exists. Partial results on both are established.

1 Preliminaries

Fuchs (see [14] and [13, Problem 1.9]), in extending an investigation by Wall [20], asked the following question: If N is any positive integer and $N = g_1 + g_2 + \cdots + g_n$ is any fixed partition of n, is it possible to find a quasigroup Q of order N which contains subquasigroups of orders $g_1, g_2, \ldots, , g_n$ whose set-theoretic union is Q? Although the answer can easily be seen to be negative for many partitions, the determination of the integer partitions for which such a quasigroup partition exists remains very far from complete. Indeed, as we shall see, Fuchs's problem is settled in general only when the integer partition contains at most two distinct integers and, consequently, when the partition has at most four parts. We begin by casting the problem in a different vernacular.

A *doubly incomplete transversal design* $DITD(k, n, (h_1 : m_1) \cdots (h_s : m_s))$ is a

1. a set V of kn points;
2. a partition $\mathcal{G} = \{G_1, \ldots, G_k\}$ of V into k *groups* of n points each;

3. a set $\mathcal{H} = \{H_1, \ldots, H_s\}$ of s *holes* so that $H_i \subset V$ for $1 \le i \le s$, $H_i \cap H_j = \emptyset$ for $1 \le i < j \le s$, and $|H_i \cap G_\ell| = h_i$ for $1 \le i \le s$ and $1 \le \ell \le k$; and

4. a set $\mathcal{M} = \{M_1, \ldots, M_s\}$ of s *subholes* so that $M_i \subset H_i$ for $1 \le i \le s$, and $|M_i \cap G_\ell| = m_i$ for $1 \le i \le s$ and $1 \le \ell \le k$; and

5. a set $\mathcal{B}$ of k-element subsets of V (*blocks*) so that every $\{x, y\} \subset V$ with $x \ne y$

   (a) is a subset of G_ℓ for some $1 \le \ell \le k$ and appears in no block;

   (b) is a subset of H_i for some $1 \le i \le s$ and appears in no block;

   (c) is a subset of $\bigcup_{i=1}^{s} M_i$ and appears in no block; or

   (d) none of the above and appears in exactly one block.

The *type* of the DITD is $(h_1 : m_1) \cdots (h_s : m_s)$, and the *side* is n.

A DITD$(k, n, (h_1 : m_1) \cdots (h_s : m_s))$ is

- a *holey incomplete transversal design* HITD$(k, (h_1 : m_1) \cdots (h_s : m_s))$ when $n = \sum_{i=1}^{s} h_i$;

- an *incomplete transversal design* ITD$(k, n, h_1 \cdots h_s)$ when $m_1 = \cdots = m_s = 0$;

- a *holey transversal design* HTD$(k, h_1 \cdots h_s)$ when $n = \sum_{i=1}^{s} h_i$ and $m_1 = \cdots = m_s = 0$; and

- a *transversal design* TD(k, n) when $s = 0$.

Exponential notation for the type ($(h_1 : m_1) \cdots (h_s : m_s)$ or $h_1 \cdots h_s$) is often used. In particular, $g_1^{i_1} g_2^{i_2} \ldots g_n^{i_n}$ denotes i_1 occurrences of g_1, i_2 occurrences of g_2 and so on. We shall use existence results for ITDs and TDs from [8] without restating them here.

These combinatorial objects can be viewed as partial latin squares when $k = 3$. More precisely, consider a $DITD(3, n, (h_1 : m_1) \cdots (h_s : m_s))$ with groups $G_1 = \{\rho_1, \ldots, \rho_n\}$, $G_2 = \{\gamma_1, \ldots, \gamma_n\}$, and $G_3 = \{\sigma_1, \ldots, \sigma_n\}$. Then form an $n \times n$ array L so that whenever $\{\rho_r, \gamma_c\}$ appears in a block with σ_ℓ, we set $L(r, c) = \ell$; if no such block exists, then $L(r, c)$ is empty. The s holes ensure that there are s disjoint empty subarrays of sides $h_1, \ldots, h_s$ on the main diagonal. In this setting, an ITD$(3, n, h_1 \cdots h_s)$ is an *incomplete latin square* of side n and type $h_1 \cdots h_s$; an HTD$(3, h_1 \cdots h_s)$ is a *partitioned incomplete latin square*; and a TD$(3, n)$ is a *latin square* of side n.

Because the empty subarrays can always be filled with latin squares, an incomplete latin square of side n and type $h_1 \cdots h_s$ is equivalent to a latin square of side n having disjoint subsquares of sides $h_1, \ldots, h_s$.

Fuchs's problem can be restated as follows: For which types $h_1 \cdots h_n$ does an HTD$(3, h_1 \cdots h_n)$ exist? We typically assume that the holes are presented in nonincreasing order by size; this can be done without loss of generality.

1.1 Known sufficient conditions

When there are few holes, the answer is known precisely:

Theorem 1.1 [16] *Let $h_1, \ldots, h_n$ be integers with $h_1 \geq h_2 \geq \cdots \geq h_n > 0$. An HTD$(3, h_1 \cdots h_n)$*

1. *always exists when $n = 1$;*
2. *never exists when $n = 2$;*
3. *exists when $n = 3$ if and only if $h_1 = h_2 = h_3$;*
4. *exists when $n = 4$ if and only if $h_1 = h_2 = h_3$, or $h_2 = h_3 = h_4$ and $h_1 \leq 2h_4$.*

In each of these cases, there are at most two different hole sizes. When there is only one hole size, we have

Theorem 1.2 [14] *When $n \geq 1$, $n \neq 2$, and $g \geq 1$, an HTD$(3, g^n)$ exists.*

Heinrich [16] considered the general case with two different hole sizes and established:

Theorem 1.3 [16] *For $u \geq v \geq 0$, an HTD$(3, u^a v^{n-a})$ exists whenever $n \geq 3$, $a \neq 2$, $n - a \neq 2$, and if $a = 1$ then $u \leq (n-2)v$.*

Kuhl and Schroeder [18] completed the determination for two hole sizes:

Theorem 1.4 [18] *For $u, v > 0$, an HTD$(3, u^2 v^{n-2})$ exists whenever $n \geq 5$, and either $u \leq v$ or $v \leq u \leq (n-2)v$.*

1.2 Variations on the theme

Let X be a set of $N = \sum_{i=1}^{n} h_i$ points. Suppose that there is an HTD$(3, h_1 \cdots h_n)$ on points $X \times \{1, 2, 3\}$ with groups $G_i = X \times \{i\}$ for $1 \leq i \leq 3$. If whenever $\{(x, 1), (y, 2), (z, 3)\}$ is a block, so also is $\{(x, 2), (y, 1), (z, 3)\}$, the corresponding partial latin square is *symmetric*. If whenever $\{(x, 1), (y, 2), (z, 3)\}$ is a block, so also is $\{(x, i), (y, j), (z, \ell)\}$ whenever $\{i, j, \ell\} = \{1, 2, 3\}$, the corresponding partial latin square is *totally symmetric*. In the latter case, we recover a well-studied combinatorial object.

A *group divisible design* (or *GDD*) is a triple $(X, \mathcal{G}, \mathcal{B})$, satisfying:

1. $\mathcal{G}$ is a partition of X into subsets (*groups*).
2. $\mathcal{B}$ is a set of subsets (*blocks*) of X such that a group and a block contain at most one common point.
3. every pair of points from distinct groups occurs in a unique block.

The *group type* of a GDD$(X, \mathcal{G}, \mathcal{B})$ is the multiset $\{|G| : G \in \mathcal{G}\}$. Again we often use exponential notation to describe group types. A GDD is referred to as a K-GDD when $|B| \in K$ for every $B \in \mathcal{B}$, and more simply as a k-GDD when $K = \{k\}$. A K-PBD (*pairwise balanced design*) is a K-GDD with all groups of size 1.

A totally symmetric HTD$(3, h_1 \cdots h_n)$ is exactly a 3-GDD of group-type $h_1 \cdots h_n$. The analogue of Fuchs's problem is: When does a 3-GDD of group-type $h_1 \cdots h_n$ exist? This problem too is far from a general solution. Indeed even the case when only two distinct group sizes arise is not settled; see [9] for the most general result, and [4, 6, 10, 12] for particular cases. Colbourn [5] develops general necessary conditions and establishes their sufficiency when $\sum_{i=1}^{n} h_i \leq 60$, but opines that there are likely further necessary conditions.

The symmetric analogue of Fuchs's problem seems not to have been studied explicitly, and the problem for 3-GDDs (the totally symmetric analogue) severely restricts the possible parameters. Nevertheless, we can use the methods of [5] to establish necessary conditions.

1.3 Basic necessary conditions

Now we return to the problem of Fuchs.

.....jnmofigklh
.....omlnhfigjk
.....ljkmnghfoi
.....kojlmnfhig
.....nkojghmifl
lkjmn....ecodab
mjnkl....abdoce
kolnm....bdcjea
nmklj....oeacbd
fnhiocbad...egm
gfmoiadec...bhn
oigfhmenb...adc
hlojgecdkiab..f
igfhkdlbacoe..j
jhigfbacedmnlk.

Table 1: An HTD$(3, 5^1 4^1 3^1 2^1 1^1)$. Holes are indicated by entries ".".

The situation for five and more holes is more complex than simply dealing with two different hole sizes. Indeed Table 1 demonstrates that all five hole sizes can be different.

Even stating the necessary conditions succinctly for five or more holes is elusive. We establish two general necessary conditions.

Lemma 1.5 *If an HTD$(3, h_1 \cdots h_n)$ exists with $h_1 \geq \cdots \geq h_n$, then $h_1 \leq \sum_{i=3}^{n} h_i$.*

Proof. In the partitioned incomplete latin square, the $h_1 \times h_2$ subarray with rows indexed by the first hole and columns indexed by the second hole contains only symbols from the third through last holes. Each column of the subarray contains distinct symbols, so $h_1 \le \sum_{i=3}^{n} h_i$. ■

Lemma 1.6 *If an HTD$(3, h_1 \cdots h_n)$ exists, then for every $D \subseteq \{1, \dots, n\}$ and $\overline{D} = \{1, \dots, n\} \setminus D$,*

$$2 \sum_{\substack{\{i,j\} \subseteq D \\ i<j}} h_i h_j + 2 \sum_{\substack{\{i,j\} \subseteq \overline{D} \\ i<j}} h_i h_j \ge \sum_{i \in D} \sum_{j \in \overline{D}} h_i h_j. \quad (1)$$

Equivalently for every subset D,

$$\left(\sum_{i=1}^{n} h_i \right)^2 - \sum_{i=1}^{n} h_i^2 \ge 3 \left(\sum_{i \in D} h_i \right) \left(\sum_{j \in \overline{D}} h_j \right), \quad (2)$$

Proof. Each block containing a pair with an entry from the ith hole and one from the jth hole with $i \in D$ and $j \in \overline{D}$ must contain two pairs intersecting both D and $\overline{D}$ and one pair contained within the holes of D or $\overline{D}$. Because

$$\left(\sum_{i=1}^{n} h_i \right)^2 = \sum_{i=1}^{n} h_i^2 + 2 \sum_{\substack{i,j \in D \\ i<j}} h_i h_j + 2 \sum_{\substack{i,j \in \overline{D} \\ i<j}} h_i h_j + 2 \sum_{\substack{i \in D \\ j \in \overline{D}}} h_i h_j,$$

inequality (2) follows. ■

While Lemma 1.6 treats 2^{n-1} different choices for the partition $\{D, \overline{D}\}$ for (2), each has the same left hand side. So it suffices to treat a single inequality in which $\sum_{i \in D} h_i$ and $\sum_{j \in \overline{D}} h_j$ are as equal as possible.

Lemmas 1.5 and 1.6, taken together, are not sufficient. To see this, observe that whenever a type $g_1^1 \cdots g_n^1$ satisfies $g_1 = g_2 = \sum_{i=3}^{n} g_i$, an HTD$(3, g_1^1 \cdots g_n^1)$ contains an HTD$(3, g_3^1 \cdots g_n^1)$. Although $10^2 6^1 4^1$ meets the condition of Lemma 1.5 it fails to meet that of Lemma 1.6 because $900 - 100 - 100 - 36 - 16 < 3 \cdot 14 \cdot 16$. On the other hand, although $10^2 2^1 1^7$ meets the condition of Lemma 1.6 (because $29^2 - 10^2 - 10^2 - 2^2 - 7 \cdot 1^2 = 630 = 3 \cdot 14 \cdot 15$), it fails to meet the condition of Lemma 1.5.

Now consider the partition $29^2 10^2 2^1 1^7$. Both Lemma 1.5 and 1.6 are met with equality, so the basic necessary conditions permit this type. Nevertheless, existence of an HTD$(3, 29^2 10^2 2^1 1^7)$ necessitates the existence of an HTD$(3, 10^2 2^1 1^7)$, which violates Lemma 1.5. Hence there are further necessary conditions, but adding a condition to eliminate the specific example given seems unlikely, by itself, to yield the true necessary conditions.

1.4 Two conjectures

We therefore focus on two cases for which we believe that the basic necessary conditions are sufficient, in the process stating two conjectures that appear to be more tractable than the entire problem.

Lemma 1.7 *If $h_1, \dots, h_n$ are integers with $n \geq 3$ for which $h_1 = h_2 = h_3 \geq \cdots \geq h_n > 0$, then the conditions of Lemmas* 1.5 *and* 1.6 *are met.*

Proof. Lemma 1.5 is always met because $h_1 = h_3 \leq \sum_{i=3}^{n} h_i$.

To establish that the conditions of Lemma 1.6 are met, consider the set D for which $\sum_{i \in D} h_i$ and $\sum_{j \in \overline{D}} h_j$ are as equal as possible. If $a, b \in D$ or $a, b \in \overline{D}$ have $0 < h_b \leq h_a < h_1$, replacing by hole sizes $h_a + 1$ and $h_b - 1$ decreases the left hand side of (2) while leaving the right hand side unchanged. So we can restrict to cases in which the hole sizes in D contain at most one hole of size not equal to h_1, and the same for the hole sizes in $\overline{D}$. Without loss of generality, the hole sizes for D are $h_1^{\alpha} h_n^1$ and for $\overline{D}$ are $h_1^{\beta} h_{n-1}^1$. Substituting $h_n + \gamma$ for h_n changes the left hand side of (2) by $2\gamma\alpha h_1$ and the right hand side by $\gamma(\beta h_1 + h_{n-1})$. When $2\alpha h_1 \leq \beta h_1 + h_{n-1}$, set $\gamma = h_1 - h_n$; otherwise set $\gamma = -h_n$. Then the inequality (2) with $h_n + \gamma$ in place of h_n implies the inequality using h_n. So we can suppose that D contains no holes of size different from h_1.

When $\alpha = \beta$, decrementing h_n changes the left hand side of the inequality of Lemma 1.6 by $-2\alpha h_1$ and the right hand side by $-\alpha h_1 - h_{n-1}$; hence we can take $h_n = 0$. Remove h_n and rewrite the partition in the form above (after which $\alpha \neq \beta$). If $\alpha > \beta$, the partition obtained by moving n to $\overline{D}$ is more equal unless $\alpha = \beta + 1$ and $h_{n-1} = h_1$. In this case, decrementing h_n by 1 changes the left hand side of the inequality of Lemma 1.6 by $-2\alpha h_1$ and the right hand side by $-\alpha h_1$; hence we can take $h_n = 0$, so that the overall hole sizes are $h_1^{2\alpha}$. If $\alpha < \beta$, the partition obtained by moving $n-1$ to D is more equal unless $\alpha = \beta - 1$ and $h_n = h_{n-1} = h_1$, so that the overall hole sizes are $h_1^{2\beta+1}$. By Lemma 1.9, h_1^n meets the condition of Lemma 1.6 when $n \geq 5$, and the verification when $n \in \{3, 4\}$ is routine. ∎

Conjecture 1.8 *Whenever $n \geq 3$ and $h_1 = h_2 = h_3 \geq \cdots \geq h_n \geq 0$ are nonnegative integers, an HTD$(3, h_1 \cdots h_n)$ exists.*

Lemma 1.9 *If $h_1, \dots, h_n$ are positive integers with $n \geq 5$ for which $h_1 \geq \cdots \geq h_n$ and $h_1 \leq (n-2)h_n$, then the conditions of Lemmas* 1.5 *and* 1.6 *are met.*

Proof. Lemma 1.5 is always met because $h_i \geq \frac{1}{n-2} h_1$ for $3 \leq i \leq n$ and hence $h_1 \leq \sum_{i=3}^{n} h_i$.

Next we show that the conditions of Lemma 1.6 are met. Consider the set D for which $\sum_{i \in D} h_i$ and $\sum_{j \in \overline{D}} h_j$ are as equal as possible. If $a, b \in D$ or $a, b \in \overline{D}$ have $h_n < h_b \leq h_a < (n-2)h_n$, replacing by hole sizes $h_a + 1$ and $h_b - 1$ decreases the left hand side of (2) while leaving the right hand side unchanged. So we can restrict to cases in which the hole sizes in D contain at most one hole of size not in $\{h_n, (n-2)h_n\}$, and the same for the hole sizes in $\overline{D}$.

D and $\overline{D}$ each contain a hole of size not in $\{h_n, (n-2)h_n\}$: Let the hole sizes in D be $((n-2)h_n)^{a_1} x^1 h_n^{b_1}$ and those in $\overline{D}$ be $((n-2)h_n)^{a_2} y^1 h_n^{b_2}$ with $h_n < x, y < (n-2)h_n$. Replacing x by $x + \alpha$, the left hand side of (2) changes by $2\alpha(a_1(n-2)+b_1)h_n$ while the right hand side changes by $\alpha((a_2(n-2)+b_2)h_n+y)$. When $2(a_1(n-2)+b_1)h_n \leq (a_2(n-2)+b_2)h_n + y$, choose $\alpha = (n-2)h_n - x$;

otherwise choose $\alpha = -(x - h_n)$. Then the inequality with $x + \alpha$ in place of x is at least as tight.

$\overline{D}$ contains a hole of size not in $\{h_n, (n-2)h_n\}$, D does not: Let the hole sizes in D be $((n-2)h_n)^{a_1} h_n^{b_1}$ and those in $\overline{D}$ be $((n-2)h_n)^{a_2} y^1 h_n^{b_2}$ with $h_n < y < (n-2)h_n$. Replacing y by $y + \alpha$, the left hand side of (2) changes by $2\alpha(a_2(n-2)+b_2)h_n$ while the right hand side changes by $\alpha((a_1(n-2)+b_1)h_n)$. When $2(a_2(n-2)+b_2) \leq a_1(n-2)+b_1$, choose $\alpha = (n-2)h_n - y$; otherwise choose $\alpha = -(y - h_n)$. Then the inequality with $y + \alpha$ in place of y is at least as tight.

When D and $\overline{D}$ contain only holes of sizes in $\{h_n, (n-2)h_n\}$, apply Theorem 1.3 or 1.4. ■

Conjecture 1.10 *Whenever $n \geq 5$ and $(n-2)h_n \geq h_1 \geq \cdots \geq h_n$ are positive integers, an HTD$(3, h_1 \cdots h_n)$ exists.*

In what follows, we describe some progress on each of these conjectures. For the first, we explore cases with few holes; for the second we explore cases in which the largest hole is not much larger than the smallest. For both, we employ standard constructions, which we introduce next.

2 Constructions

We use a number of constructions, most simple and some more complex. We state them without proof, because they pervade the literature on GDDs, PBDs, and TDs.

2.1 Filling a Hole

Lemma 2.1 *If an HTD$(3, h_1 \cdots h_t)$ exists and an HTD$(3, g_1 \cdots g_r)$ exists with $h_t = \sum_{i=1}^{r} g_i$, then an HTD$(3, h_1 \cdots h_{t-1} g_1 \cdots g_r)$ exists.*

2.2 Filling a GDD

Let $(V, \mathcal{G}, \mathcal{B})$ be a K-GDD of type $h_1 \cdots h_\ell$; let $\mathcal{G} = \{G_1, \ldots, G_\ell\}$ with $|G_i| = h_i$ for $1 \leq i \leq \ell$. Let $w : V \mapsto \mathbb{Z}_{\geq 0}$ be a weight function. Suppose that, for every $B = \{v_1, \ldots, v_k\} \in \mathcal{B}$, an $HTD(3, \sum_{i=1}^{k} w(v_i), w(v_1) \cdots w(v_k))$ exists. Then there exists an

$$HTD(3, \sum_{x \in V} w(x), (\sum_{x \in G_1} w(x)) \cdots (\sum_{x \in G_\ell} w(x))).$$

2.3 Inflation

Giving weight α to each point and using a TD$(3, \alpha)$ for each block, we obtain:

Lemma 2.2 *Suppose that a $DITD(3, n, (h_1 : m_1) \cdots (h_s : m_s))$ exists. Then when $\alpha \geq 1$ is an integer, a $DITD(3, n, (\alpha h_1 : \alpha m_1) \cdots (\alpha h_s : \alpha m_s))$ exists.*

2.4 The Wilson TD construction

We use varations of a construction of Wilson [21]; see [3, 7] for general results in this direction. We simply list three variants here.

Theorem 2.3 *Let k, r, n, m be integers. Let $V = \{1, \ldots, n\} \times \mathbb{Z}_n$. Let $m \in \mathbb{Z}_+$ and $w : \{k+1, \ldots, k+r\} \times X \mapsto \mathbb{Z}_{\geq 0}$ (a* weight *function). Suppose that there exist*

1. *a 'master' $TD(k+r, n)$, $(V, \mathcal{G}, \mathcal{B})$, with $\mathcal{G} = \{G_1, \ldots, G_{k+r}\}$ where $G_\ell = \{\ell\} \times \mathbb{Z}_n$ for $1 \leq \ell \leq k+r$;*
2. *an $ITD(k, m + \sum_{\ell=k+1}^{k+r} w(z_\ell), w(z_{k+1}) \cdots w(z_{k+r}))$ for every $B = \{z_1, \ldots, z_{k+r}\} \in \mathcal{B}$, with $B \cap G_\ell = \{z_\ell\}$ for $1 \leq \ell \leq k+r$.*

Then an $ITD(k, mn + \sum_{\ell=k+1}^{k+r} \sum_{i=0}^{n-1} w((\ell, i)), [\sum_{i=0}^{n-1} w((k+1, i))] \cdots [\sum_{i=0}^{n-1} w((k+r, i))])$ exists.

Theorem 2.4 *Let k, r, n, m be integers. Let $V = \{1, \ldots, n\} \times \mathbb{Z}_n$. Let $m \in \mathbb{Z}_+$ and $w : \{k+1, \ldots, k+r\} \times X \mapsto \mathbb{Z}_{\geq 0}$ (a* weight *function), where $w((k+r, n-1)) = 0$. Suppose that there exist*

1. *a 'master' $TD(k+r, n)$, $(V, \mathcal{G}, \mathcal{B})$, with $\mathcal{G} = \{G_1, \ldots, G_{k+r}\}$ where $G_\ell = \{\ell\} \times \mathbb{Z}_n$ for $1 \leq \ell \leq k+r$, and $\mathcal{B}_1 = \{\{(i, j) : 1 \leq j < k+r\} \cup \{(n-1, k+r)\} : i \in \mathbb{Z}_n\} \subset \mathcal{B}$;*
2. *an $ITD(k, m + \sum_{\ell=k+1}^{k+r} w(z_\ell), w(z_{k+1}) \cdots w(z_{k+r}))$ for every $B \in \mathcal{B} \setminus \mathcal{B}_1$, $B = \{z_1, \ldots, z_{k+r}\}$ with $B \cap G_\ell = \{z_\ell\}$ for $1 \leq \ell \leq k+r$;*
3. *for $k < \ell < k+r$, an $HTD(k, w((\ell, 0)) \cdots w((\ell, n-1)))$.*

Then an $HTD(k, [m + \sum_{\ell=k+1}^{k+r-1} w((\ell, 0))] \cdots [m + \sum_{\ell=k+1}^{k+r-1} w((\ell, n-1))][\sum_{i=0}^{n-2} w((k+r, i)])$ exists.

Theorem 2.5 *Let $k, \ell, h_1, \ldots, h_s$ be integers and $n = \sum_{i=1}^{s} h_i$. Let $X = \{(i, j) : 1 \leq i \leq s, 1 \leq j \leq h_i\}$, and $V = \{1, \ldots, k\} \times X$. Let $m \in \mathbb{Z}_+$ and $w : \{k+1, \ldots, k+r\} \times X \mapsto \mathbb{Z}_{\geq 0}$ (a* weight *function). Suppose that there exist*

1. *a 'master' $HTD(k+r, h_1 \cdots, h_s)$, $(V, \mathcal{G}, \mathcal{H}, \mathcal{B})$, with $\mathcal{G} = \{G_1, \ldots, G_{k+r}\}$ where $G_\ell = \{\ell\} \times X$ for $1 \leq \ell \leq k+r$, and $\mathcal{H} = \{\{(\ell, i, j) : 1 \leq \ell \leq k, 1 \leq j \leq h_i\} : 1 \leq i \leq s\}$;*
2. *an $ITD(k, m + \sum_{\ell=k+1}^{k+r} w(z_\ell), w(z_{k+1}) \cdots w(z_{k+r}))$ for every $B = \{z_1, \ldots, z_{k+r}\} \in \mathcal{B}$, with $B \cap G_\ell = \{z_\ell\}$ for $1 \leq \ell \leq k+r$;*
3. *an $HTD(k, \sum_{i=1}^{s} \sum_{j=1}^{h_i} w((\ell, i, j)), [\sum_{j=1}^{h_1} w((\ell, 1, j))] \cdots [\sum_{j=1}^{h_s} w((\ell, s, j))])$, for $k < \ell \leq k+r$.*

Then an $HTD(k, [mh_1 + \sum_{\ell=k+1}^{k+r} \sum_{j=1}^{h_1} w((\ell, 1, j))] \cdots [mh_s + \sum_{\ell=k+1}^{k+r} \sum_{j=1}^{h_s} w((\ell, s, j))])$ exists.

3 Small hole sizes

In order to provide needed ingredients in the constructions of later sections, we first explore ITDs and HTDs in which all hole sizes are quite small.

Lemma 3.1 *An* ITD$(3, b+\sum_{i=1}^{r} g_i, g_1 \cdots g_r)$ *with* $b \geq g_1 \geq \cdots \geq g_r \geq 0$ *exists when* $r \leq 2$ *or* $b \leq 2$.

Proof. Without loss of generality, $g_r > 0$. When $r = 0$, the required ITD is a latin square or TD$(3, b)$. When $r = 1$, the required ITD is a latin square of side $b + g_1$ with a sub-square of side g_1 [15]. Suppose henceforth that $r \geq 2$. Using Lemma 2.2, we can suppose that $\gcd(b, g_1, \ldots, g_r) = 1$. We treat the small cases for b.

$b = 1$**:** Then $g_1 = \cdots = g_r = 1$ and an HTD$(3, 1^{r+1})$ exists by Theorem 1.2; fill one hole to get the required ITD.

$b = 2$**:** Suppose that $g_1 = \cdots = g_s = 2$ and $g_{s+1} = \cdots = g_r = 1$. An HTD$(3, 2^{s+1}1^{r-s})$ exists by Theorem 1.3 or 1.4. Similarly, an HTD$(3, 2^s 1^{2+r-s})$ exists by Theorem 1.3 or 1.4..

Now we treat cases with $r = 2$ and $b > 2$. A TD$(5, b)$ exists when $b \notin \{2, 3, 6, 10\}$. Apply Theorem 2.3 using $m = 1$ and all weights in $\{0, 1\}$ to treat all remaining cases when $b \notin \{3, 6, 10\}$. Apply Theorem 2.3 using $m = 2$, $n = 5$, and all weights in $\{0, 1, 2\}$ to treat all remaining cases when $b = 10$. Using Lemma 2.2, we handle all cases when $\gcd(b, g_1, g_2) > 1$. Filling holes in 3-HTDs of types 3^3, $1^4 3^1$, $2^3 1^1$, $2^1 1^4$, and 1^5 when $b = 3$, and of types $6^1 1^7$, $5^3 1^1$, $5^1 3^3$, $5^1 2^4$, $5^1 1^7$, $4^1 3^3$, $4^1 1^7$, $3^1 2^4$, $3^1 1^7$, $2^1 1^7$, and 1^8 when $b = 6$, leaves only the case ITD$(3, 3+3+2, 3^1 2^1)$ for $b = 3$, and the ITD$(3, 6+6+5, 6^1 5^1)$ and ITD$(3, 6+5+4, 5^1 4^1)$ with $b = 6$. For the first, use Theorem 2.4 on a TD$(5, 4)$ with weight 1 to form an HTD$(3, 3^1 2^1 1^3)$. For the latter two, use Theorem 2.4 on a TD$(8, 7)$ with weight 1 to form an HTD$(3, 6^1 5^1 1^6)$ and an HTD$(3, 5^1 4^1 1^6)$. Then fill all holes of size 1 in each. ■

Lemma 3.2 *Suppose that an* HTD$(3+r, 1^n)$ *exists. Let* $\{x_i : 1 \leq i \leq n\}$ *be integers with* $r \geq x_1 \geq \cdots \geq x_n \geq 0$ *for which* $x_1 + x_2 \leq r + \sum_{i=3}^{n} x_i$. *Then an* HTD$(3, n + \sum_{i=1}^{n} x_i, (x_1 + 1) \cdots (x_n + 1))$ *exists.*

Proof. Apply Theorem 2.5 with $m = 1$ and $k = 3$. The weight function is determined as follows. Let $\tau = \max(x_3, x_1 + x_2 - r)$. Assign weight 1 to each point in $H_i \cap G_j$ when $3 < j \leq 3 + \tau$ and $i \in \{1, 2\}$; to each point in $H_1 \cap G_j$ for $3 + \tau < j \leq 3 + x_1$; and to each point in $H_2 \cap G_j$ for $3 + x_1 < j \leq 3 + x_1 + x_2 - \tau$. In addition, for $3 \leq i \leq n$, give weight 1 to x_i points in $H_i \cap (\cup_{j=4}^{\tau} G_j)$ in such a way that each group G_j with $4 \leq j \leq \tau$ contains at least 1 point of weight 1 from $\cup_{i=3}^{n} H_i$. The ingredient ITDs are ITD$(3, s, 1^{s-1})$ for $1 \leq s \leq r$ from Lemma 3.1, and the ingredient HTDs are HTD$(3, s, 1^s)$ for $3 \leq s \leq n$ from Theorem 1.2. ■

Lemma 3.3 *An* HTD$(3, 3^2 2^1 1^2)$, *an* HTD$(3, 4^1 3^1 2^1 1^3)$, *an* HTD$(3, 4^1 3^2 1^2)$, *and an* HTD$(3, 4^1 3^2 2^1 1^2)$ *exist.*

Proof.

Type $3^2 2^1 1^2$
...hjidegf
...ghjfide
...jghifed
hij...cabg
ihg...bjca
gji...achb
edfiba..jc
jedacb..fi
fgebacjd.h
dfhcigeba.

Type $4^1 3^1 2^1 1^3$
....lijekfgh
....jhlkgife
....hkigfelj
....ilkfjheg
hjik...cblda
jikl...bdahc
khli...jadcb
lkejdcb..gaf
elfgbac..kjd
feghcbadl.ik
gfjeadhlcb.i
ighfkjdaecb.

Type $4^1 3^2 1^2$
....ljigfkhe
....jkhfelig
....hijlkgef
....ilkegfjh
jlhi...cdbak
hijl...kbdca
lhik...bcadj
fklecba...gd
kegfacd...lb
efkgdal...bc
gjfhbdcale.i
igejkhbdacf.

Type $4^1 3^2 2^1 1^2$
....jinklehgfm
....imkeglfhnj
....khimnfgjle
....lkjnemifgh
jikn...lcambhd
mlnh...dabjcik
lhmj...akdbnci
fkeganc...dmbl
kegfclm...nadb
ngfmbal...edkc
gmjihcbfdn..ea
hniemjdbfc..ag
ijhlndagbkce.f
eflkdbhcmgaij.

■

Theorem 3.4 *For $n \geq 5$, there exists an HTD$(3, g_1 \cdots g_n)$ whenever* $\min(n-2, 4) \geq g_1 \geq g_2 \geq \cdots \geq g_n \geq 1$.

Proof. **$n = 5$:** Apply Lemma 3.2 to an HTD$(5, 1^5)$ to treat all cases with $g_1 + g_2 \leq 1 + \sum_{i=3}^{5} g_i$. Apply Lemma 2.1 to an HTD$(3, 3^3)$ to produce an HTD$(3, 3^2 1^3)$. Apply Theorem 2.4 with $k = 3$ and $m = 1$ to a TD(5,4) to produce an HTD$(3, 3^1 2^1 1^3)$. The final HTD$(3, 3^2 2^1 1^2)$ is from Lemma 3.3.

$n = 6$: To treat cases with $g_2 \leq 3$, apply Theorem 2.4 with weight 1 to a TD$(6, 5)$. If $g_2 + g_3 \leq 1 + \sum_{i=4}^{6} g_i$ or $g_3 = 1$, give g_1 points in the last group weight 1, and distribute weights on the remaining five groups as in Lemma 3.2. So we can suppose that $g_2 = 3$. If $g_1 = 3$ and $g_3 + g_4 + g_5 \geq 5$, give g_6 points in the last group weight 1, and distribute weights on the remaining five groups as in Lemma 3.2. When $g_2 \leq 3$, the remaining cases, an HTD$(3, 4^1 3^2 2^1 1^2)$ and an HTD$(3, 4^1 3^1 2^1 1^3)$, are from Lemma 3.3.

Now we must treat cases with $g_1 = g_2 = 4$. Start with a 5-GDD of type 4^6 with groups $G_1, \ldots, G_6$.

1. Truncate two groups to form a $\{3, 4, 5\}$-GDD of type $4^4 a^1 b^1$ for $1 \leq b \leq a \leq 4$.
2. Choose a block B that misses G_6. Let $\{y\} = B \cap G_5$ and $\{z\} = B \cap G_4$. Let $x \in G_5$ with $x \neq y$. Let w be the point in G_6 that lies on the block containing x and z. Then delete y, z, and any subset of G_6 to form a $\{3, 4, 5\}$-GDD of type $4^3 3^2 a^1$ for $0 \leq a \leq 4$. Instead delete $\{x, y, z\}$ and any subset of $G_6 \setminus \{w\}$ to form a $\{3, 4, 5\}$-GDD of type $4^3 3^1 2^1 a^1$ for $1 \leq a \leq 4$.

3. Choose a block B that misses G_1. Delete the points of B in $G_3, \ldots, G_6$ and any further subset of G_6 to form a $\{3,4,5\}$-GDD of type $4^2 3^3 a^1$ for $0 \le a \le 3$.

Now start with a 5-GDD of type 4^5 with groups $G_1, \ldots, G_5$. Let $x \in G_5$, and $B_1, \ldots B_4$ be the blocks containing x.

1. Delete all points in G_5 except for x. Delete any subset of points in G_4. Then blocks $B_1, \ldots B_4$ have size 4 or 5, while all other blocks have size 3 or 4. Fill each block B other than $B_1, \ldots B_4$ with an $\mathrm{HTD}(3, 1^{|B|})$. Give weight 2 to x and fill each $B \in \{B_2, B_3, B_4\}$ with an $\mathrm{HTD}(3, 2^1 1^{|B|-1})$. Finally fill B_1 with an $\mathrm{HTD}(3, 1^{|B_1|+1})$. This creates an $\mathrm{HTD}(3, 4^3 a^1 1^2)$ for $0 \le a \le 4$.
2. Delete all points in G_5 except for x. Delete the points in $B_1 \cap G_3$ and $B_1 \cap G_4$. Proceed as before to form an $\mathrm{HTD}(3, 4^2 3^2 1^2)$.

Fill a hole in an HTD(3,4^3) to obtain an $\mathrm{HTD}(3, 4^2 1^4)$. For $\mathrm{HTD}(3, 4^s 2^{6-s})$ with $0 \le s \le 6$, apply Lemma 2.2. For the remaining cases, see Table 2.

Type $4^2 3^1 1^3$
....inmlhgekfj
....kmjilfnhge
....jlnkfhgiem
....miljnehfkg
knjl....amcbid
ilkn....madjcb
jkim....dblcna
ljni....bcmdak
ghledcam...nbf
nefgcbda...mlh
egmfndcb...ahl
fmehbkidgna.jc
hfgjlakncdbe.i
mihkajbcelfgd.

Type $4^2 3^1 2^1 1^2$
....kmjnflhigoe
....ilojefmnkhg
....moilnhgejkf
....jimkhenofgl
nkji....mabdolc
imkl....bnoacdj
kjon....amlcdib
jlno....cbdkiam
lgmfonbd...heca
molgdacb...fneh
ofehlbdc...gamn
fhijnkaogce..bd
hegkbcnadof..ji
gifecjlmodabh.k
enhmadkilgcjbf.

Type $4^2 3^1 2^2 1^1$
....jplieofghkmn
....koilnegjpmhf
....ojmnfhlkigpe
....linkgfmhepjo
jpik....olandcbm
mijp....bncokdla
pjml....acndobki
knlo....pmdajicb
lmoedbac...pnhfg
olpncabm...fgedh
hgnmpdco...bfael
ghefnkobcap..jid
iekhbnpjdgo..fac
efgimljdhpbca..k
fkhgamdplbeic..j
nofjickamdheblg.

Type $4^2 3^1 2^3$
....nqpihoefgkljm
....omjnfgqkepilh
....jimopegqkhfnl
....mplkohfnjiqeg
lqkp....nmojabcid
kpni....dqlabmjco
oilq....cbadnjpmk
pkim....alncoqbdj
eomldbcq...pfghan
mnphcloa...eqdgbf
fmhnqdbc...oplage
jhoeinqpgdb..ckfa
nfjgpkidqch..aeob
qjefkadblpmgi..hc
geqjbcalmfphd..ki
ilgoajkmencbhfd..
hgfklonjbadicem..

Type $4^2 2^1 1^3$
....mjikhlegf
....ikjlfghme
....limjkfgeh
....jlkmehfig
klim....cbdaj
limk....dabjc
mjli....adckb
jmkl....bcadi
ekhgadcb..mfl
hgefbmda..lck
fhgjdclime.ba
iefhkbacgmj.d
gfjecabdlkih.

Type $4^2 2^2 1^2$
....njkilfhgem
....lkijmefngh
....mljnekghfi
....inlmfhjekg
klnj....acdmib
jkmn....blciad
ijlk....nmbdca
lijm....cnabdk
fehgkcbd..manl
ghkeamdb..nflc
hnficbmadg..je
emgfdincha..bj
ngehbaclkdij.f
mfiljdakgbech.

Type $4^2 2^3 1^1$
....iklneomfjgh
....kmoilejgfhn
....lnkomhgjeif
....jimlfkhngoe
ijlm....ocnakdb
nikl....adcobjm
lkno....dmaicbj
jlok....bnimacd
mnhgaodc..belfk
khgfncbm..edola
homnbacjgf..dei
gmfeojidnb..hac
efjidbakhgoc..l
oeihcdjaklfb..g
fgejmlnbcadhik.

Type $4^2 3^2 2^1 1^1$
....olpngemqfikjh
....lknqomehijfgp
....pmjkfhgieqnlo
....mqolnphjgkife
mkli....qodpabjcn
kiqm....lcoabpdnj
njpq....ablkdocim
pnij....blqcoamkd
fpmonabd...gcehql
olngcpab...ehdqmf
qhoebnma...fpgldc
gojpadkchqf...ebi
ifkhjcqopdb...gea
jegkqodicfp...ahb
lgfndjimeacbqh..k
hqefibljmnadkc..g
emhlkicpdgnojfba.

Type $4^3 2^2 1^1$
....lmnjeofqgikhp
....komphqenigljf
....nkplqegofjhim
....opjnfmhelkgqi
iqjk....mnpbaocld
lkqo....banpcdimj
jipq....ncombldka
knli....obqajpmdc
ngofpdbm....qeach
pemgdcaq....ohfnb
mpneabdo....hcqfg
gfhpmqcb....daneo
fhklqiodgpbc..jae
eofjclqipdah..bgk
hmenbjiacfdgkq..l
qjgmialkdhcfeb..n
olihjnkcagmdpfeb.

Type $4^2 3^2 2^2$
....rqpiflekogmnhj
....pmnrqfhikoegjl
....oiljmnqprefhgk
....kojqehngpfrlmi
qlnk....aopjcrbidm
jmlq....prdaibkcon
ikmj....lqrodpnacb
noil....cmaqbkjrpd
lfohmprn...caqdebg
mrpodnbc...ehagqlf
ogqrbcma...dehlfnp
pqrgijokbaf...hdec
hpefarkdgbc...qjio
rjhickqpodg...abfe
fnkejlcbdgmrqi..ah
ehjnqbimrclfgd..ka
kigpnadlheobfjcm..
gefmldaonpbhjcik..

Table 2: Small cases for Theorem 3.4

$n = 7$**:** Apply Lemma 3.2 to an $\mathrm{HTD}(7, 1^7)$ to treat all cases with $g_1 + g_2 \le 1 + \sum_{i=3}^{7} g_i$. The remaining types are $4^2 1^5$, $4^2 2^1 1^4$, and $4^1 3^1 1^5$. Fill holes in an $\mathrm{HTD}(3, 4^3 a^1)$ for $a \in \{1, 2\}$ to produce the first two. For the last, fill a hole of size 3 in an $\mathrm{HTD}(3, 4^1 3^2 1^2)$ from Lemma 3.3.

$n \geq 8$, $n \notin \{10, 14, 18, 22\}$: An HTD$(3, g_1 \cdots g_5 1^3)$ is obtained by filling a hole of size 3 in an HTD$(3, g_1 \cdots g_5 3^1)$. Apply Lemma 3.2 to an HTD$(6, 1^n)$ to treat all remaining cases.

$n \in \{10, 14, 18, 22\}$: Apply Theorem 2.4 to a TD$(7, n-1)$ to handle the remaining cases.

This completes the proof. ■

Lemma 3.5 *An ITD$(3, b+\sum_{i=1}^{r} g_i, g_1 \cdots g_r)$ with $b \geq g_1 \geq \cdots \geq g_r \geq 0$ exists when $b \leq 4$.*

Proof. Without loss of generality, $g_r > 0$. When $r \leq 2$ or $b \leq 2$, apply Lemma 3.1. Otherwise, fill holes in an HTD$(3, 1^b g_1^1 \cdots g_r^1)$ from Theorem 3.4. ■

Lemma 3.6 *Suppose that a TD(n, g) exists. Let $g_1, \ldots, g_n$ be nonnegative integers with $g_1 \geq \cdots \geq g_n$.*

1. *If $n \geq 5$, $g = g_1 = g_2$, $g - 1 = g_3$, and $\sum_{i=4}^{n} g_i \geq g$ then an HTD$(3, g_1^1 \cdots g_n^1)$ exists.*
2. *If $n \geq 5$, $g + 1 = g_1$, $g = g_2 = g_3$, and $\sum_{i=4}^{n} g_i \geq g$ then an HTD$(3, g_1^1 \cdots g_n^1)$ exists.*

Proof. Suppose that the TD(n, g) has groups $G_1, \ldots, G_n$. Choose a point $x \in G_3$. For the first statement, give weight 0 to x and weight 1 to all points in $G_1, G_2, G_3 \setminus \{x\}$. On each of the g blocks through x, ensure that at least one point has weight 1 in the groups $G_4, \ldots, G_n$. Then employ HTD$(3, 1^s)$ for $s \geq 3$ to produce the required HTD. For the second statement, give weight 2 to x and weight 1 to all points in $G_1, G_2, G_3 \setminus \{x\}$. On each of the g blocks through x, ensure that at least one point has weight 1 in the groups $G_4, \ldots, G_n$. Then employ HTD$(3, 2^1 1^s)$ and HTD$(3, 1^s)$ for $s \geq 3$ to produce the required HTD. ■

Now we treat the first cases in Conjecture 1.8.

Lemma 3.7 *When $n \in \{5, 6\}$ and $g_1 = g_2 = g_3 \geq g_4 \cdots \geq g_n \geq 0$, there exists an HTD$(3, g_1^1 \cdots g_n^1)$.*

Proof. When $n = 5$ and $g_1 \leq 3$, or $n \geq 6$ and $g_1 \leq 4$, apply Theorem 3.4. When $n = 5$ and $g_1 = 4$, truncate two groups of a TD(5,4) and give weight 1. Except when $g_1 \in \{6, 10, 14, 18, 22\}$, form a TD$(6, g_1)$. Truncate three groups and give weight 1 to handle all cases with $g_1 = g_2 = g_3 \geq g_4 \cdots \geq g_6 \geq 0$. When $g_1 \in \{10, 14, 18, 22\}$, form a TD$(6, g_1/2)$. Give weight 2 to all points in three groups and weights from $\{0, 1, 2\}$ to all points in the remaining three to handle all cases with $g_1 = g_2 = g_3 \geq g_4 \cdots \geq g_6 \geq 0$.

It remains only to handle cases when $g_1 = 6$. Start with a TD(6,5) with groups $G_1, \ldots G_6$. Let B be a block of the TD. On the points of B, give weight 2 in the first three groups and weights from $\{0, 1, 2\}$ in the remaining three. On the points not in B, give weight 1 in the first four groups and weights from $\{0, 1\}$ in the remaining two. This handles all cases with $g_4 \geq 4$. When $g_4 = 3$ and $g_5 + g_6 \geq 3$, let $y \in G_4 \setminus B$. In groups G_1, G_2, G_3, give weight 2 to the points of B and 1 to the remaining points. In groups G_4, G_5, G_6, give weight 0 or 1 to the points of B. Give y weight 0 and all points of $G_4 \setminus B$ other than y weight 1. Let $\{z_i\} = G_i \cap B$ for $1 \leq i \leq 3$. For $1 \leq i \leq 3$, ensure that the block that contains y and z_i also contains a point of weight 1 in $G_5 \cup G_6$. Give weight 0 or 1 to each of the remaining points.

When $g_4 + g_5 + g_6 \leq 4$, give weight 2 to three groups of a TD(4,3). On one point y of the fourth group, give weight $\sum_{i=4}^{6} g_i$; on the others give weight 0. Let B be a block containing y. Fill blocks not containing y using HTDs of type 2^3, those containing y other than B using type $2^3(\sum_{i=4}^{6} g_i)^1$, and B with an HTD of type $2^3 g_4^1 g_5^1 g_6^1$.

For types $6^3 3^1 2^1$, $6^3 3^1 1^2$, and $6^3 2^2 1^1$, use the following:

Type $6^3 3^1 2^1$	Type $6^3 3^1 1^2$	Type $6^3 2^2 1^1$
......svntowjkgiulhmrqp	......qrmwvousihtlpjkgn	......porqsnljkutivmwgh
......wtronmlvjhskiqpug	......sopvqugktilhjnwrm	......uqnmprtwjgskloihv
......qnpwmtsjuglvkhoir	......tqnsmrwgujivhkopl	......qnupmvgktsljwihro
......tomrvpkhlugsqwijn	......usopnqivwtgklrjmh	......wmtvoujlikhgnqpsr
......upsmqrgitwjhlovnk	......wpuotnljkgvsmqihr	......ouqrnpshwlvtigmjk
......pmwqrsulhkijovngt	......nuvmwpshglkjiortq	......mrpuqshtljgwkvnoi
uonmrw......bfctvepdqas	mtorqn......vwbscdfpaue	porumt......ebsvwdqafcn
rpunvq......adsetwmcfbo	onmtsr......cdeabuqvpwf	vqotwn......fuaecsprdbm
tqmvor......dsacwunpbef	voqntw......fudbsercmap	orsmpv......deftaubnqwc
mrtons......cuvaedfbwpq	prnmos......atvcuwdfqeb	ntqwrm......bfvdecousap
ouqrsp......fwevbcdnatm	qmpvrt......decuwfbanso	tmunsr......vcewfbapoqd
snrqpu......vcbfdtwemoa	tprunm......bfsdecowvqa	mnpotq......adcfuvrwbes
kjwshibdavtu......efgcl	ghjivueasbct......wldfk	jughvitescwf......dlakb
jtghwkvsubfd......caeli	iugshvabtfew......cdlkj	lkisguctvbfw......jheda
wgluihfctdae......vjksb	hktwujcvaedf......gibls	gwtiusevcdba......hklfj
lwsjtvabduef......gickh	jlsgwkvtbcfd......aehiu	slvkjhbwatue......cdgif
ishtkgdwecbv......aljfu	usijklfwdtav......ehcbg	iswjhgvcdatb......ufkle
hviguteafswc......bkldj	kwulihbecdsa......vgfjt	kghvilsafwdc......ebjtu
vkjwmnofqapbegdlhi...rc	njvqgimcwrpbelfkha...od	qvjrkwdbonemuaghif..cpl
nmkplorqvecawbijfg...hd	wglpjormqabekchfdi...nv	uhnqlpadwfcokvbije..rmg
qhvlgjcropdniawbkf...me	lvwkmqpfrnochbjeag...di	wimgqknpeordcshablfj..t
gipkqlmecnuohtfsabjrd..	sqkhlpodeurmjiawftnbg.c	rjlpnofsmeatwidbkhgc..q
ploijmnubfsqtekdcargh..	rihopgdnfqustalvjbkmec.	hpklojrfbsvqigucdametn.

This completes the proof. ■

4 Some GDDs and MGDDs

We also require some known results on PBDs, GDDs, and MGDDs. First we define the latter. A *K-modified GDD*, or K-MGDD($a \times b$), is a set of ab points, equipped with a parallel class of blocks of size a, a parallel class of blocks of size b, and all other blocks of sizes from K, so that every two distinct points occur together in exactly one block. The two parallel classes form the *first groups* and the *second groups*.

Lemma 4.1 *Let n be a positive integer. Suppose that for a set K of positive integers, and a positive integer m,*

1. *a K-GDD of type s^n exists; and*
2. *whenever $k \in K$ and $3m \geq j_1 \geq \cdots \geq j_k \geq m$, an HTD$(3, j_1^1 \cdots j_k^1)$ exists.*

Then an HTD$(3, g_1^1 \cdots g_n^1)$ exists whenever $3ms \geq g_1 \geq \cdots \geq g_n \geq ms$.

Proof. Give weights from $\{m, \dots, 3m\}$ to the points of the GDD. For each block of size k, the weights assigned in the block lie between m and $3m$, and hence the weighted block can be filled with an HTD. ■

Lemma 4.2 *Let s and n be positive integers. Let K and L be sets of positive integers. Suppose that a L-PBD of order s exists and that a K-MGDD$(\ell \times n)$ exists for each $\ell \in L$. Then a K-MGDD$(s \times n)$ and a $K \cup \{n\}$-GDD of type s^n exist.*

Proof. Suppose that V is the point set of the PBD. We form the K-MGDD on $V \times \{1, \dots, n\}$. For each block B of the PBD, we place a K-MGDD on $B \times \{1, \dots, n\}$, aligning first groups on $B \times \{i\}$ for $1 \leq i \leq n$ and second groups on $\{x\} \times \{1, \dots, n\}$ for $x \in B$. To produce the GDD, place a block on each of the second groups. ■

Lemma 4.3 *Let $5 \leq n \leq 24$ and*

1. *$\ell \in \{5, 7, 8, 9, 11, 13, 17, 19, 23, 29, 30, 31, 43, 102, 107\}$ if $n = 5$;*
2. *$\ell \in \{6, 7, 8, 9, 11, 13, 17, 19, 23, 29, 37, 41, 47, 101, 137, 149, 167\}$ if $n \in \{6, 10\}$.*
3. *$\ell \in \{5, 6, 8, 9, 11, 13, 17, 19, 23, 29, 47, 67, 79, 83, 103, 107, 119\}$ if $n = 20$.*
4. *$\ell \in \{5, 6, 7, 8, 9, 11, 13, 17, 19, 23, 29\}$ otherwise.*

Then a K-MGDD$(\ell \times n)$ exists with $K \subseteq \{5, \dots, \min(n, \ell)\}$.

Proof. If $\alpha \geq 0$, $\ell+\alpha$ is a prime power, and $5+\alpha \leq n \leq \ell+\alpha$, the MGDD is obtained from an HTD$(n, 1^{\ell+\alpha})$ by deleting all points in α holes. Similarly if $\beta \geq 0$, $n + \beta$ is a prime power and $n + \beta \geq \ell + \beta \geq 5$, the MGDD is obtained from an HTD$(\ell, 1^{n+\beta})$ by deleting all points in β holes. A 5-MGDD$(\ell \times 5)$ (and a 5-MGDD$(5 \times \ell)$) for $\ell \in \{12, 14, 15, 18, 20, 21, 22, 30, 102\}$ is from an HTD$(5, 1^\ell)$. A 6-MGDD$(6 \times \ell)$ for $\ell \in \{20, 21\}$ is from an HTD$(6, 1^\ell)$. A $\{5, 6\}$-MGDD(6×14) is obtained from an HTD$(6, 1^{15})$ by deleting one hole. ■

Lemma 4.4 *There exist*

1. *5-GDDs of types 1^{21}, 2^{21}, and 4^n whenever $n \equiv 0, 1 \pmod 5$;*
2. *$\{5, 9\}$-GDDs of type 4^n when $n \in \{14, 19\}$;*
3. *$\{5, 13\}$-GDDs of type 4^n when $n \in \{13, 18, 23\}$;*

4. *$\{5, 17\}$-GDDs of type 4^n when $n \in \{17, 22, 24\}$;*

5. *a 6-GDD of type 5^6;*

6. *a $\{7, 8, 19\}$-GDD of type 7^{20}; and*

7. *a $\{5, 6, 7, 8, 9, 10\}$-GDD of type 5^{10}.*

Proof. The first four are from [1, 2]. The 6-GDD of type 5^6 is a TD(6,5). The $\{7, 8, 19\}$-GDD of type 7^{20} is obtained by truncating a group of a TD(8,19) to seven points. The $\{5, 6, 7, 8, 9, 10\}$-GDD of type 5^{10} is obtained from a TD(10,9) by deleting all points on four blocks through one point to get a $\{6, 7, 8, 9, 10\}$-GDD of type $5^9 8^1$ and then deleting three further points from the group of size 8. ■

5 Largest group at most three times the smallest

Conjecture 1.10 asserts that whenever there are n holes, and the largest hole size does not exceed $n - 2$ times the size of the smallest, the HTD exists. We examine the situation when the largest is no more than three times the smallest here.

To avoid repetition, we define two statements:

$\mathbb{S}_{n,\mu}$: Whenever $3\mu \geq g_1 \geq g_2 \geq \cdots \geq g_n \geq \mu$, an HTD($3, g_1 \cdots g_n$) exists.

$\mathbb{P}_{n,\mu}$: Whenever $3\mu \geq g_1 \geq g_2 \geq \cdots \geq g_n = \mu$ and $g_{n-2} > g_n$ when $n \geq 7$, an HTD($3, g_1 \cdots g_n$) exists.

Our concern is to establish $\mathbb{S}_{n,\mu}$, but we focus on $\mathbb{P}_{n,\mu}$. Theorem 3.4 establishes the statement $\mathbb{S}_{n,1}$ for all $n \geq 5$.

Lemma 5.1 *Suppose that*

1. *$\mathbb{P}_{m,\mu}$ holds whenever $m \geq 5$, $\mu \geq 1$, and $(m, \mu) \neq (5, 6)$;*

2. *Whenever $18 \geq g_1 \geq g_2 \geq g_3 \geq g_4 = g_5 = g_6 = g_7 = 6$, an HTD$(3, g_1 \cdots g_7)$ exists; and*

3. *Whenever $3\theta \geq g_1 \geq g_2 \geq \cdots \geq g_5 = 6 > \theta$, an HTD$(3, g_1 \cdots g_5)$ exists.*

Then $\mathbb{S}_{n,\theta}$ holds for all $n \geq 5$, $\theta \geq 1$, and $(n, \theta) \neq (5, 6)$.

Proof. If $n \geq 7$ and $g_{n-2} = g_{n-1} = g_n$, by $\mathbb{P}_{n-2,g_{n-3}}$ there is a solution for type $g_1^1 \cdots g_{n-3}^1 (3g_n)^1$ except possibly when $n = 7$ and $g_4 = 6$; when $n = 7$ and $g_7 < 6$, there is a solution using the third supposition in the statement of the lemma with $\theta = g_7$. In either case fill the hole using an HTD$(3, g_n^3)$. The remaining cases are treated by the second supposition.

If $g_n > \mu$, $\mathbb{S}_{n,\mu}$ follows from statements $\mathbb{P}_{n,g_n}$ for $\mu \leq g_n \leq 3\mu$ except when $(n, g_n) = (5, 6)$, and $\mu \in \{2, 3, 4, 5, 6\}$. The exceptional cases are treated by the third supposition. ■

Henceforth we can focus on $\mathbb{P}_{m,\mu}$, also handling the two exceptional situations in Lemma 5.1.

Lemma 5.2 *Suppose that $\mathbb{S}_{m,\mu}$ holds whenever $25 \geq m \geq 5$, $\mu \geq 1$, and $(m,\mu) \neq (5,6)$. Then $\mathbb{P}_{n,\theta}$ holds (and hence $\mathbb{S}_{n,\theta}$ holds) for all $n \geq 26$ and $\theta \geq 1$.*

Proof. Let $\alpha = \lfloor \log_5(n-1) \rfloor$. We proceed by induction on α. When $\alpha \leq 1$, the statement of the lemma dictates that $\mathbb{P}_{n,\mu}$ holds (and hence $\mathbb{S}_{n,\mu}$ holds) whenever $(n,\mu) \neq (5,6)$. We suppose that the statement is true when $\alpha < \tau$, and prove it for $\alpha = \tau$. In general, we proceed as follows. Suppose that $\{g_1,\dots,g_n\}$ can be partitioned into five classes $G_1,\dots,G_5$, setting $\mu_i = \sum_{g \in G_i} g$ for $1 \leq i \leq 5$, so that $\max(\mu_i : 1 \leq i \leq 5) \leq 3\min(\mu_i : 1 \leq i \leq 5)$, $|G_i| \geq 5$ for $1 \leq i \leq 5$, and $|G_i| \geq 6$ whenever $\min(|G_i| : 1 \leq i \leq n) = g_n = 6$. Note that $\mu_i \geq 10$ because $g_n > 1$. First use $\mathbb{S}_{5,\min(\mu_i:1\leq i\leq 5)}$ (which holds by induction) to produce an HTD$(3, \mu_1^1 \cdots \mu_5^1)$, T. Then use $\mathbb{S}_{|G_i|,\min(|G_i|)}$ when $g_n = 6$, or $\mathbb{S}_{|G_i|,g_n}$ when $g_n \neq 6$, which both hold by induction; fill the groups of T using HTD$(3, G_i)$ for $1 \leq i \leq 5$.

We must produce the required partition of $\{g_1,\dots,g_n\}$ into classes $G_1,\dots,G_5$. First partition into classes each containing $\lfloor \frac{n}{5} \rfloor$ or $\lceil \frac{n}{5} \rceil$ of the $\{g_i\}$ so that $\max(\mu_i : 1 \leq i \leq 5) - \min(\mu_i : 1 \leq i \leq 5)$ is as small as possible. This produces a class of size 5 only when $26 \leq n \leq 29$, and hence completes the proof except when $26 \leq n \leq 29$ and $g_n = 6$. Because we need only establish $\mathbb{P}_{n,6}$ for $26 \leq n \leq 29$, we may assume that $g_{n-2} > 6$.

Partition $\{g_1,\dots,g_n\}$ into classes $G_1,\dots,G_5$ so that $5 \leq |G_i| \leq |G_j| \leq 6$ whenever $1 \leq i < j \leq 5$, and when $g_i \in G_a$ and $g_j \in G_b$, we have $a \leq b$ when $i < j$. Then none of $G_1,\dots,G_4$ contains an entry equal to 6, and $|G_5| = 6$, so this partition completes the proof. ■

Lemma 5.3 *Suppose that $\mathbb{P}_{m,\tau}$ holds whenever $(m,\tau) \neq (5,6)$ and $\tau \in X_n$ (from Table 3). Then*

1. *$\mathbb{P}_{n,\mu}$ holds whenever $n \geq 5$, $\mu \geq 1$, and $(n,\mu) \neq (5,6)$;*
2. *Whenever $18 \geq g_1 \geq g_2 \geq g_3 \geq g_4 = g_5 = g_6 = g_7 = 6$, an HTD$(3, g_1 \cdots g_7)$ exists; and*
3. *Whenever $3\theta \geq g_1 \geq g_2 \geq \cdots \geq g_5 = 6 > \theta$, an HTD$(3, g_1 \cdots g_5)$ exists.*

Proof. According to Lemma 5.2 we need only treat cases with $n \leq 25$. Consider the first statement. In Table 3, we examine choices for g_n for each $n \leq 25$. For each n, let R_n be all positive integers not in $G_n \cup B_n \cup M_n \cup U_n \cup X_n$. When $\ell \in R_n$, a B_n-PBD of order ℓ exists [11, 19]. By Lemma 4.3, a K-MGDD$(\ell \times n)$ with $K \subseteq \{5,\dots,\min(n,\ell)\}$ exists for each $\ell \in B_n \cup M_n$. Hence applying Lemma 4.2 we obtain a K-MGDD$(\ell \times n)$ and a K-GDD of type ℓ^n with $K \subseteq \{5,\dots,\min(n,\ell)\}$ for each $\ell \in B_n \cup M_n \cup R_n$. Next using Lemma 4.4 we obtain a K-GDD of type ℓ^n with $K \subseteq \{5,\dots,n\}$ for each $\ell \in B_n \cup M_n \cup R_n \cup G_n$. One can verify that every entry in U_n can be written as $m\kappa$ for some $\kappa \in B_n \cup M_n \cup R_n \cup G_n$, with $m \neq 6$ when $n = 5$.

n	G_n	B_n	M_n	U_n	X_n
5	4	**5 7 8 9**	11 13 17 19 23 29 30 31 43 102 107	10 12 14 15 16 18 20 22 24 26 27 28 32 33 34 38 39 42 44 46 51 52 60 94 95 96 98 99 100 104 106 108 110 111 116 138 140 142 146 150 154 156 158 162 166 170 172 174 206	*1 2 3 6*
6, 10	4,5	**6 7 8 9**	11 13 17 19 23 29 37 41 47 101 137 149 167	10 12 14 15 16 18 20 21 22 24 25 26 27 28 30 32 33 34 35 36 38 39 40 45 46 93 94 95 98 99 100 138 139 142 143 144 145 146 147 148 150 152 153 154 155 160 161 166 185	*1 2 3*
7–9, 12		**5 6 7 8 9**	11 13 17 19 23 29	10 12 14 15 16 18 20 22 24 27 28 32 33 34	*1 2 3 4*
11, 13–19, 22–24	4	**5 6 7 8 9**	11 13 17 19 23 29	10 12 14 15 16 18 20 22 24 27 28 32 33 34	*1 2 3*
20	4,7	**5 6 8 9**	11 13 17 19 23 29 47 67 79 83 103 107 119	10 12 14 15 16 18 20 22 24 27 28 32 33 34 35 38 39 42 53 82 84 87 92 98 99 118 122 123 124 142 172 182	*1 2 3*
21,25	1,2				

Table 3: Reducing to a finite number of values for g_n

Hence applying Lemma 4.1 we produce an HTD$(3, g_1^1 \cdots g_n^1)$ whenever $5 \leq n \leq 25$ and $g_n \notin X_n$.

The second statement follows by using a 5-GDD of type 5^5. The third statement follows by using a $\{6, 7\}$-GDD of type 6^7. ■

Lemma 5.4 $\mathbb{P}_{n,4}$ *holds whenever* $n \geq 5$.

Proof. By Lemma 5.3 we need only consider $n \in \{7, 8, 9, 12\}$. When $n \in \{7, 8, 9\}$, apply Theorem 2.5 to an HTD(7,1^n) with $m = r = 4$, using ITDs from Lemma 3.5. To produce the needed HTDs, let $x_i = g_i - 4$ and $y_i = \max(0, x_i - 4)$ for $1 \leq i \leq n$. Let ℓ be the largest index for which $x_\ell \geq 4$. Recall that $x_5 > 0$. Let κ be the largest index for which $y_\kappa > 0$. Table 4 specifies the HTDs to be used, which all exist by Lemma 3.7 or Theorem 3.4.

Finally, to produce an HTD$(3, 12^2 5^3 4^2)$, we produce an HTD$(3, 15^1 12^2 4^2)$ and fill a hole. The required HTD is obtained from a TD(5,4), giving weight 1 to all points in two groups, weight 3 to all points in two further groups, and weight 3 or 4 to each point in the final group. All ingredients are obtained from Theorem 3.4 except the HTD$(4^1 3^2 1^2)$ from Lemma 3.3.

When $n = 12$, apply Theorem 2.5 to an HTD(6,1^{12}) with weight 4. Now $x_{10} > 1$. If $x_2 \geq 2$, write $x_i = a_i + b_i$ so that $a_i, b_i \leq 4$, and at least six of $\{a_1, \ldots, a_{12}\}$

Case	3-HTD # 1	3-HTD # 2	3-HTD # 3	3-HTD # 4
$\ell \geq 3$, $\kappa = 0$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	–	–	–
$\ell \geq 3$, $\kappa = 1$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	y_1^1	–	–
$\ell \geq 3$, $\kappa = 2$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	y_1^1	y_2^1	–
$\ell \geq 3$, $\kappa \in \{3,4\}$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	$y_3^3 y_4^1$	$(y_1 - y_3)^1$	$(y_2 - y_3)^1$
$\ell \geq 3$, $\kappa = 5$, $y_3 = 4$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	$4^3 y_4^1 \cdots y_\kappa^1$	–	–
$\ell \geq 3$, $\kappa = 5$, $y_3 < 4$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	$y_3^3 y_4^1 y_5^1$	$(y_1 - y_3)^1$	$(y_2 - y_3)^1$
$\ell \geq 3$, $\kappa \geq 6$	$4^\ell x_{\ell+1}^1 \cdots x_n^1$	$y_1^1 \cdots y_\kappa^1$	–	–
$\ell = 2$, $x_2 \leq 7$	$3^2 x_3^1 \cdots x_n^1$	$(x_2 - 3)^1$	$(x_2 - 3)^1$	$(x_1 - x_2)^1$
$\ell = 2$, $x_1 = x_2 = 8$, $x_3 \geq 2$	$3^2 (x_3 - 1)^1 x_4^1 \cdots x_n^1$	1^3	4^1	4^1
$\ell = 1$	$2^1 x_2^1 \cdots x_n^1$	$\lceil (x_1 - 2)/2 \rceil^1$	$\lfloor (x_1 - 2)/2 \rfloor^1$	–
$\ell = 0$	$x_1^1 \cdots x_n^1$	–	–	–

Table 4: HTDs for applications of Theorem 2.5 with $m = 4$

and $\{b_1, \ldots, b_{12}\}$ are nonzero. Then 3-HTDs of types $a_1^1 \cdots a_{12}^1$ and $b_1^1 \cdots b_{12}^1$ exist by Theorem 3.4. So suppose that $x_2 = 1$. Then 3-HTDs of types $\lfloor x_1/2 \rfloor^1$, $\lceil x_1/2 \rceil^1$, and $0^1 x_2^1 \cdots x_{12}^1$ exist. ■

Lemma 5.5 *If $\mathbb{P}_{m,\mu}$ holds whenever $m \in \{5, 6\}$ and $\mu \in \{2, 3\}$, $\mathbb{P}_{n,\theta}$ holds for all $n \geq 5$ and $\theta \geq 1$ except possibly when $(n, \theta) = (5, 6)$.*

Proof. First apply Lemmas 5.3 and 5.4. So suppose that $g_n = \mu \in \{2, 3\}$, $\mu \in X_n$, and $n \geq 7$. Using weight g_n, apply Theorem 2.5 to an HTD$(5, 1^n)$ when $n \notin \{6, 10\}$, or Theorem 2.4 to a TD$(6, n-1)$ when $n \in \{6, 10\}$. Let $\nu = n$ in the first case, and $\nu = n-1$ in the second. The required ITDs are all from Lemma 3.5. Let $x_i = g_i - g_n$ for $1 \leq i \leq \nu$. Table 5 specifies the HTDs to be used, which all exist by Theorem 3.4.

Case	3-HTD # 1	3-HTD # 2
$x_5 \geq 2$	$\lceil x_1/2 \rceil \cdots \lceil x_\nu/2 \rceil$	$\lfloor x_1/2 \rfloor \cdots \lfloor x_\nu/2 \rfloor$
$x_5 = 1$, $x_4 > g_n$	g_n^4	$(x_1 - g_n) \cdots (x_4 - g_n) x_5 \cdots x_\nu$
$x_5 = 1$, $x_3 > g_n \geq g_4$	g_n^3	$(x_1 - g_n) \cdots (x_3 - g_n) x_4 \cdots x_\nu$
$x_5 = 1$, $x_1 > g_n \geq g_2$	g_n^1	$(x_1 - g_n) x_2 \cdots x_\nu$
$x_1 \geq x_2 > g_n \geq x_3$, $x_5 = \cdots = x_8 = 1$	$\lfloor x_1/2 \rfloor \lfloor x_2/2 \rfloor x_6 x_7 x_8$	$\lceil x_1/2 \rceil \lceil x_2/2 \rceil x_3 x_4 x_5 x_9 \cdots x_\nu$

Table 5: HTDs for applications of Theorem 2.5 with weights two and three

In this way, all cases with $x_8 \geq 1$ are handled. When $n \geq 10$, no remaining cases have $x_8 = 0$, and so the determination is complete. When $n \in \{7, 8, 9\}$, apply Theorem 2.5 to an HTD$(n, 1^n)$ to treat the remaining cases. ■

Theorem 5.6 *If $n \geq 5$, $\mu \geq 1$, and $3\mu \geq g_1 \geq g_2 \geq \cdots \geq g_n \geq \mu$, then an HTD(3,$g_1 \cdots g_n$) exists except possibly when $n = 5$ and $\mu = 6$.*

Proof. Lemmas 5.5 and 5.3, together with Theorem 3.4, handle all cases except when $n \in \{5, 6\}$ when $g_n \in \{2, 3\}$. If $\gcd(g_1, \dots, g_n) > 1$, apply Lemma 2.2.

$n = 5$, $g_n = 2$: When $g_3 = g_4 = g_5 = 2$, apply Theorem 2.4 to a TD(5,4) with weight 2 to handle all cases. Apply Lemma 3.6(1) to a TD(5,4). Apply Theorem 2.5 to an HTD(5,1^5) with weight 2, using HTDs from Theorems 1.1, 3.7, and 3.4. The remaining cases (66532 66522 66432 66332 66322 65522 65442 65432 65422 65332 65322 64332 64322 63322 55422 55332 55322 54322) are given in Appendix 1.

$n = 6$, $g_n = 2$: If $g_1 = g_2 = 6$ and $g_4 = g_5 = g_6 = 2$, fill a hole in an HTD$(3, 6^3 g_3^1)$. If $g_1 \leq 4$, apply Theorem 3.4. Apply Lemma 3.6(1) and (2) to a TD(6,5). Apply Theorem 2.4 to a TD(6,5) using weight $\min(4, g_{n-1})$, using HTDs from Theorems 1.1, 3.7, and 3.4. The remaining cases (664332 665322 664322 663322 654322 654222 653222 553222) are given in Appendix 2.

$n = 5$, $g_n = 3$: Apply Theorem 2.4 to a TD(5,4) with weight 3 to handle all cases when $g_3 = g_4 = g_5 = 3$, or when $6 \geq g_2 = g_3 = g_4 \geq 3$. Apply Lemma 3.6(1) to a TD(5,5). Apply Theorem 2.5 to an HTD(5,1^5) with weight 2 to handle types $6^2 4^2 3^1$ and $6^1 5^1 4^1 3^2$. Apply Theorem 2.5 to an HTD(5,1^5) with weight 3, using HTDs from Theorems 1.1, 3.7, and 3.4. The remaining cases (99843 99833 99743 99733 99643 99543 99443 99433 98843 98833 98773 98763 98743 98733 98663 98643 98633 98543 98443 98433 97763 97743 97733 97663 97643 97633 97543 97433 96543 96443 96433 95543 95443 95433 94433 88743 88733 88643 88633 88543 88443 88433 87733 87663 87643 87633 87543 87443 87433 86543 86443 86433 85443 85433 84433 77633 77543 77443 77433 76543 76443 76433 75433 66433) are given in Appendix 3.

$n = 6$, $g_n = 3$: If $g_1 = g_2 = 9$ and $g_4 = g_5 = g_6 = 3$, fill a hole in an HTD$(3, 9^3 g_3^1)$. Apply Lemma 3.6(1) to a TD(6,8) and a TD(6,9), 3.6(2) to a TD(6,7) and a TD(6,8). When $g_1 \leq 8$, apply Theorem 2.4 to a TD(6,5) using weight 2 to handle types $8^1 6^1 5^1 4^1 3^2$, $7^1 6^1 5^1 4^1 3^2$, $8^1 6^1 4^1 3^3$, and $6^2 4^2 3^2$. Apply Theorem 2.4 to a TD(6,5) using weight 3 or $\min(4, g_{n-1})$, using HTDs from Theorems 1.1, 3.7, and 3.4. The remaining cases (997433 996443 996433 995443 995433 994433 987733 987633 987433 987333 986443 986433 986333 985443 985433 984433 984333 977633 977433 977333 976433 976333 975433 974433 974333 965433 964433 955433 886433 886333 885443 885433 884433 884333 876633 876433 876333 875433 874333 775433 774333) are given in Appendix 4.

This completes the proof. ■

6 Concluding remarks

At the present time, a complete solution to the problem of Fuchs appears to be out of reach. In part this results from the large number and variety of different partitions into hole sizes, and the insufficiency of the basic necessary conditions. Rather than restricting partitions to few different hole sizes, we have focussed on two somewhat general situations in which the basic necessary conditions are met, and appear to be sufficient.

First we have examined cases in which the largest hole size is "not too much larger than" the smallest. Conjecture 1.10 asserts that when there are $n \geq 5$ holes, and the largest is no more than $n - 2$ times as large as the smallest, a solution to Fuchs's problem exists. When there are $n = 5$ holes, the largest can be no more than three times the smallest. We show that Conjecture 1.10 holds when $n = 5$ except possibly when the smallest hole size is equal to 6. Unfortunately this is not adequate to establish the conjecture in general, but we establish the weaker statement that when there are more than five holes and the largest is at most three times the size of the smallest, a solution exists. We expect that the great variety of solutions produced will be useful in addressing the still more general conjecture.

Secondly we have examined cases in which the three largest groups have the same size. Conjecture 1.8 asserts that in these cases, a solution always exists. We establish the conjecture when the number of holes is at least three and at most six. Again we believe that the solutions produced will be useful in addressing Conjecture 1.8 in general.

To treat Fuchs's problem in its entirety, the main concern at present is to determine the further necessary conditions. In the absence of this, we believe that Conjectures 1.8 and 1.10 are useful intermediate goals.

Note Added in Proof: Some of the results in this paper were established independently in [17]; that paper focusses on the cases with five groups, using techniques different from those employed here.

Appendix 1. $n = 5$ and $g_5 = 2$

Type $6^2 5^1 3^1 2^1$
......mnqtoujsrlvpikhg
......vqmorpkhtsjilugn
......nvrpumtglkiqjhso
......pstnmrvijuhgolqk
......orvusqgjihlnpmkt
......rupvqnikgtsjhoml
tnosuv......aderfbqcpm
orvqmp......sudbtfcena
vpmoqr......ftsauenbcd
pqtuvo......cbfermdnas
usqtpn......erbvdcmaof
smnvtq......bfudcoaper
hkulgiscbrva.....dfjte
ruhgsledactv.....kbifj
ljgrktdbeacs.....uvfih
giskjhtfdeab.....luvrc
ilkhrucasfdt.....vegjb
mojnigapudbchevfk...lq
nhijlkqocmfeuvagb...dp
kvlmnjueoqpfdahcg...bi
jtrposfmnbedlckiahgq..
qgpihmbtfsnorlcjeakd..

Type $6^2 5^1 2^2$
......munrtsjilghoqkp
......sptmoukhjignlrq
......ursnqphlgktmjio
......qnopsturhjkimgl
......ropqmnljutskghi
......nsqupotgrhjlimk
purmqn......scteabdof
qsmruo......efabdptcn
sqtonr......dbeucapfm
nmqpot......busrecfda
tpsnmq......cadfbuoer
unotsm......rdcafqepb
giuhtkfebcrd.....jals
hlksijcfrdeb.....tuag
lkijgubtafdr.....ecsh
ihlkrgtcdsua.....fbje
jrgilsadutfe.....hkbc
ogjuhpeqmabcftkli..nd
ktnlphoafbcmgeidu..qj
mohgjlpbceaqikfsrdn..
rjpqkidmeonfasbclgh..

Type $6^2 4^1 3^1 2^1$
......tnqprujghilkmos
......nmtsqrijkgopulh
......osuqpmhktrnigjl
......qprmotuljkhgisn
......pontusgqihkjlmr
......urpntoksglmhjqi
mqsotu......radcbenpf
onqtum......srbdpaefc
trumnp......ecqsfbdao
ponqrs......dteaumfcb
suonmq......cdrtafpbe
nmrpst......bufqecoda
rsjuklfebcaq....idthg
gihslrdtafeb....juckq
ujglqicaebsf....dthrk
jhkrigsqcufa....tlbed
itmkpobufdcnlhae...gj
lktighebmodcafuj...np
hlpgjkafoendtbcu...im
kgijhnmdsrbpqelfcoa..
qplhojrcdamefisbgnk..

Type $6^2 3^2 2^1$
......qprsnmghltjoik
......ntmqsolgjihkrp
......montrpqkglishj
......tropmnsqihljkg
......sqpnorilhktgjm
......ontrpqjikmshgl
otpnmr......bfseadqc
rnqmsp......tcfdoabe
pqmrto......cdbsefna
smrqpt......dbeocnaf
mpntrq......fsacbeod
nstoqm......repadcfb
hlisjgcadfqe...bktpr
tksplirfqeda...jgbch
jrhkispceabt...gfldq
ghkinlfdabtsejc...mo
logjknbmsdfchat...ei
igjhokesbmcfatd...ln
qilghjdecoabkprfnm..
kjolghabfcedprqnmi..

Type $6^2 3^1 2^2$
......qrpnoslkjmgih
......rsnpmqhjgliok
......nprosmkqijhlg
......snqmpoirhglkj
......pqmsnrjikhogl
......omsrqngplkjhi
omsqrn......dbcafpe
srnmqp......aefdbco
mnrpos......cfqedab
npormq......bcsfeda
pqmnsr......faboced
qopsnm......edrbafc
hklijgabfdec...rsqp
iljghkdeaqcp...srbf
rsglphbacedf...ikjq
lihkgjcfobaersd..nm
kgijlofcdarbshe..mn
jhkoiledbcfaqgpnm..
gjqhkimoefbdplacn..

Type $6^1 5^2 2^2$
......ptlrnsjiqhomgk
......qlmnoihrgkstjp
......lropmgkqsthjni
......mqnsrhgtijpklo
......rnsopkijtqmlhg
......nprltqskjigomh
lsromp.....dtfacnbqe
tnmsol.....fbcraedpq
nmolqt.....cfserdpab
ptqrnm.....ecdbfasol
olnmpq.....rdefstcba
kqghsrtcaeb.....ifdj
igktrhsdqba.....jefc
grikjsfbtaq.....ched
sjhqgkaectd.....birf
qhsjtgefbdc.....kair
jptihnosdflbagke..cm
mojplibafcstehdg..kn
rkpgijcoemfaqbhdln..
hilnkodmpqejracbfg..

Type $6^1 5^1 4^2 2^1$
......tqnlsikhgmjoupr
......mouslpqjtihngrk
......unltqrsphkimjog
......orqmnjpsktuglhi
......rtsqmkiuplghonj
......nurpoqhtjgmkisl
qltnpr.....abefuodcms
pnqrum.....tacsolbdfe
lmpqot.....sfbrnauedc
tuomns.....ecrqfdlabp
spmolq.....brfuetcnad
jskuqipdcer....hbtfga
gruikjsbfdp....ceatqh
rqgjsudepct....akfhib
hgstrpecabu....dfikjq
ojikghcmtnbuead....lf
nilgtofabudhjkc....em
mthljnbfeacgudi....ko
ukjhmgalofedtib....cn
ionshklpdrafgqebcjm..
khrpilqsmofcdgajneb..

Type $6^1 5^1 4^1 3^1 2^1$
......pnomqjigshltrk
......orpqnhjsitmklg
......qmtpsghjrknloi
......lqsomkrhtjingp
......tolnrpgqkshimj
......mpnstqkrhljgio
qtrmsn.....abfpocedl
mnstpl.....fqdbaocer
pltqro.....dsecbfmna
tplsom.....rfbqnedac
nmqrlp.....tacefsobd
rgpjikcsftb....edaqh
hrkijtacqbp....dgsfe
ksjgtibdref....cahpq
jqgphrsaecd....itbkf
sjokghfbmlaietd...cn
oihlngetbfcsdka...jm
gknomsdfcaletij...hb
iomhkqnldrecpafgbj..
lhinqjreadobcpgmkf..

Type $6^1 5^1 4^1 2^2$
......qnpmrjisgkhlo
......oqlsnhrkimjgp
......losnqijgphrkm
......slmqokpjrnghi
......nsrplgkhqiomj
......prnlmqgikosjh
normpq.....sdabelfc
qnsrom.....cefdlbpa
pmlsqr.....efcabnod
lrmpno.....bcdsaeqf
rsonml.....abpcfdeq
sqigjhdpfea....rcbk
jhpqksrdbaf....gice
kiqhspefcrb....jadg
ighkrjceqdp....sfab
ojglhkmaecdrsbf..in
mkjogibcafsdhre..nl
glkjinfbdoepaqhcm..
hpnilgamobcfqejdk..

Type $6^1 5^1 3^2 2^1$
......lnmrpqiksghoj
......plqsmokrghnji
......nosmrgqjhklip
......rqopnigskjmlh
......splqorjhmignk
......qrpnskogjlihm
osmqrn.....pfblcdae
lrsmpo.....aecfnbqd
mprnql.....scdaefbo
smlonq.....bdprfcea
rqnlmp.....daobsecf
pjohgsdcefa...ibrkq
jhqpoiascdb...erkfg
kigrhjeafoq...cdspb
glijskbmacehrf...dn
inhkjgfbredcsa...ml
nkjsirmdbalfhe...gc
hgkilmofnbcepqdaj..
qopgkhcedlfjbinma..

Type $6^1 5^1 3^1 2^2$
......mlnoqgjhripk
......opmrlkiqnhjg
......qnomrjkpilgh
......rmqlpihgjnko
......lqpnorgikjhm
......nrlpmhokqgij
mqporn.....bcadelf
nporql.....fadmcbe
pnqlom.....drbeafc
rlmnpo.....qefbdca
omrplq.....adcfben
qohjkpbfaec...grdi
kjighrdobfe...cqap
jkgqihpercb...afod
igkhmjeadqfcbr..nl
hrligkfdcaneqj..mb
ghjmnicbedapfolk..
linkjgacfbdopehm..

Type $6^1 4^1 3^2 2^1$
......lpqmgnhikroj
......qnploirgjkmh
......mlnqrhpkijgo
......kompjqirhgln
......omrknpqjlihg
......pqkrhonlgmji
lnpmko....qecbrdfa
nqokmp....brdealcf
mlnork....pbecfqad
omlnpr....fdaqbekc
hjgioqrcdn...feapb
iorqgjeafb...dchnp
jrhpigndof...aqcbe
rgqhjmceldafb...ik
qikrhlbfaecgj...dm
khmjqifrbcdag...el
gpjlnhakeoicfmdb..
pkiglndbcaejohmf..

Type $6^1 4^1 3^1 2^2$
......opklqjimhgn
......nkpoiqgljmh
......knmqjghipol
......pmlnohqjikg
......moqkgpnhlij
......qlopnijgmhk
knloqm....pdbefca
lmqnko....afcpdbe
pqnkml....beocadf
nomlpk....eadbqfc
jpgqohabef...dcni
qghpinbfcd...aejo
hiojnpdafc...qgeb
gkpihjcqamfbe..ld
ijkglqfdbehcp..am
ohjmgiledbcnafk..
mlihjgecnadofkb..

Type $6^1 3^2 2^2$
......mlopinghkj
......lkmihopjng
......njkophiglm
......kpnhomjigl
......ompgnihljk
......jnlmgpkohi
ojmkpl...afebcdn
nolpkj...fedcamb
kpjmno...eabldcf
mipnogcde...abfh
pngihmabc...ofed
hgiomndca...fpbe
jlkhgpeobdcf..ia
lkojihpefbdg..ac
ghnljibfdcmaek..
imhglkfajnbcde..

Type $5^2 4^1 2^2$
.....rkqnohgipmfjl
.....kmpqlrhofjing
.....lnkpmfjqrhgoi
.....pomrkgqfhlnij
.....mqrknjpgifhlo
oknml.....bdcqraep
mplok.....erabdqcn
plmrn.....qaeobdkc
lnokp.....dercqmba
nqklr.....pobeacdm
grijfqaodp....cbhe
jiqfhodace....grpb
rjfqobpdac....iegh
ihgpqabcor....ejfd
qfrhgelnbdicja..mk
kghimnrleqcbdj..af
fmpgjceblaoihdnk..
hojnidcembafpgkl..

Type $5^2 3^2 2^1$
.....kmprniqofljhg
.....oqlkrfjnghmpi
.....rpklqhoijmgnf
.....pnrmogiflkqjh
.....lrqpmogjhfkin
pnoqk.....bcrmalde
omprq.....nbcaedlk
lokmn.....adbeqrcp
nlqom.....rpekbcad
kplno.....qedcramb
qhipgndace...rjfbo
irnjfeobdp...qchga
fgrhpqcnba...dieoj
jfhgicleabdrq...km
himkrabdqljfg...ec
rqgijbkmedcha...fl
mkjfldeoncpahigb..
gjflhmacokenpbdi..

Type $5^2 3^1 2^2$
.....mlkpoqnjfhgi
.....lopnqifgjkhm
.....oqnklgifhpmj
.....pkoqmfjhgiln
.....qmlonhgipjfk
lopkq.....bdnmcea
pqonm.....abdkecl
knqmo.....cepalbd
nmklp.....oqebdac
mlnpk.....ecaqbdo
gphofncbad...iqje
qfihjbnedp...caog
ohfjiepqca...dgnb
fkgqladmecjpb..ih
ijlghcbdmepaq..kf
hijfgdeabknoclm..
jgminkaclbdhoef..

Type $5^1 4^1 3^1 2^2$
.....omjkihplgnf
.....pklnmfogjih
.....mjpofnghlki
.....lpomngifkhj
.....nlmjpihkofg
lmjno....dbcpeak
okplm....edncbja
npojk....aedbcml
klnoj....cabepdm
pfgmnaodb...ihce
hompfebac...dign
mghipbnca...ofed
ijfhldakpgoe..bc
fhikgjcedopa..lb
gnlfikdbehcmja..
jikghcenlbmfad..

Appendix 2. $n = 6$ and $g_6 = 2$

Type $6^2 4^1 3^2 2^1$
......vnomuqkxhgjplriwts
......mqxonruwltpihgjsvk
......xwsptmqliuhgvnkojr
......qrpxwsgktvijnhmluo
......rsnvouthqiwlmjgxkp
......puvqxwhjkslntorgmi
mopnwu......vdsatecqxrbf
nmqtus......crvedxofwbpa
twmvpo......acxrefbsdqnu
vptrow......bsuqamfxcedn
oxupnq......ebrwvtadscfm
sqowmr......fvcxubdanpet
jrgxlhdtcfsa....kvuwbiqe
rvjstkebucfx....ghwildaq
ujhqgtcarsdv....xwiefklb
wtrivguxedqf....bcklhasj
inlkhjadbeptxuwc...mofgv
hgkuxvtmwnbciefl...pajod
guxojinvdacpwtbf...kemhl
qhilkxopfwensgdbmaj...rc
xlsginwombaerqjkfdp...ch
lkwhspfcqrmbdaejnox...ig
pinjrmsetuvolfadckgbqh..
ksvmqlbfatrdjighouecpn..

Type $6^2 5^1 3^1 2^2$
......mtopsrkxiwluvjnqgh
......rpstvnukhlgxwiqmjo
......qutmophjrxsgivwknl
......xvwqmtgukrjihnlpos
......oqurpwihxkvjlgtsmn
......porwuqjgsitmnxkhlv
pwntsm......racveqdobxfu
noqxtr......sbvudcawmfpe
xtsmnq......vdafboucewrp
srupqo......cvwexnmfabtd
msxnrv......bwetufqpdoac
vntuop......wrdsfbemxcqa
ixrhvtuwcfds.....ebaglkj
tviglubeaxwc.....hfdjrsk
gujlihtbdsce.....wxkravf
ugvskwexbctf.....ajlidhr
ljwrxkfcedau.....vghstbi
kqpounwfmbevxljda...hicg
qkhwgxvdnubofelac...pjim
whlkmjdapvxbefgci...onuq
hlmjpisrxnfadctgwkoq..eb
rmkiwgcnfoqxasbjhlpe..dt
oigqjsnmvardltuhkpcbfe..
jpovhlasqenmtifbrdkucg..

Type $6^2 4^1 3^1 2^2$
......qtrvmpihlgkjuwson
......mwtqnrguhipojlvks
......srputvljihwgomnqk
......vpwsqmtrulonkgjhi
......pnqostrkwuhmgvijl
......rovpuqjitsnhlkwgm
qpsnwo......btfvmecdaur
mvnrou......dsctbweqfpa
wmpovn......aqerucdfbst
ormwps......qavceufndtb
vnwstq......udafcpmerbo
swounr......cfbdtvpameq
lgjtqheuscbd....viwrkaf
kiqgrjbeufwc....atvsldh
iuvqktwscrfb....dahjgle
hslkigcdetvw....jfabqru
jokvuptmndafwlgb...heic
nltmgkuvdeoahbjw...icfp
tjhilvdcbmeufwka...pong
pqihjwoafncsevrkglb..md
rhgpmlnbawdovesqfki..cj
utrlsmafobpekgqjidnch..
gkujhifqmarnscdelbtop..

Type $6^2 3^2 2^2$
......uvqrosikhtlgnpmj
......sumpvtgiqholrjnk
......qosnturglkihvmjp
......nqtumoprjvhsgikl
......rmuonvqlpjktighs
......mtpqurjhkinvolsg
smoqpn......taduvbfrce
tnqpov......cuesmdafbr
qunorp......avsefmcbdt
vprtuo......fsbcendaqm
utvmnq......sfabcepdro
rqmnst......upvadfboec
pshutjvrafde...gbcqkli
ijsrqgadevbc...ltkuhpf
klgihrbsfcpa...djuevtq
hoksliebcdafvjt...mugn
gvjlimfanscbdtu...keoh
lguhvktfobsmedc...jnia
nipvkhocdmrqlegfuj..ab
okljgucpveqnhbrmai..fd
jhikmspnrtedbqfogalc..
mrtgjldebafpkcinsohq..

Type $6^1 5^1 4^1 3^1 2^2$
......lovumjritgkspqhn
......pstmlghjivuorknq
......rpslqtuvgmhkinoj
......otlvupsgjimnhrqk
......umprtskhvnoiqljg
......mruqoivkpjglnhst
rnotpu.....fceqlsvdmab
lrmoqn.....utsfeabvpcd
monvsp.....abqcfdtluer
npumot.....qfdeslabvrc
pvquno.....cardtemfbls
ughrjqvfdbs....atecikp
turihvseqpd....kbcgjfa
qtjpridcesv....ufgkabh
jhsqtkfbrep....cvuagdi
sjgkimblnocvdah...ufte
vstlmhnubafdick...jego
givjkstafcnheub...odml
okinvlcqafergbudjh..pm
kmlhugevcdrbqpaonj..if
hlksgjqdonaeptrbifmc..
iqpglranmtbkjfshcdeo..

Type $6^1 5^1 4^1 2^3$
......mnqstrujiohpkgl
......sptloujkhgnmqri
......pumrlsthknojiqg
......utnmrqksgjlhoip
......olrtuhipqkgnmsj
......nslqmghrptiujko
mlonqu.....dctfesabpr
lmnosq.....epcurtdfab
tnqulm.....psbdarecof
pultrn.....aqfsdcoebm
nouqmp.....badrcetlfs
ghjkpsqrdbf....iuctea
jprstgefacq....ubidhk
qjkihrtcbup....safgde
irtpjkbafes....hdgucq
kgslojfmuaetric..bhnd
rsgjitdocfakeub..lnmh
htimuoadpnbfgqelk..jc
ukphgicqeodjfatbm..ln
oihgklrbspncdejmfqa..
sqmrnhleodcibgafjkp..

Type $6^1 5^1 3^1 2^3$
......oqsmngjtihklpr
......rpotqihjklsgnm
......lomnrhsgtijkqp
......snloprqkgmtijh
......psqrmtgohjlnki
......mltpsqkinrgohj
pqtrmn.....desbfoalc
lmrons.....bteadpfcq
ntlpqr.....eabmsfcod
mnostp.....afclqedrb
olpqrm.....fcdsbntae
sohkjtcfbqd...rgipea
hpsjokqbrdf...ctaeig
grktijedpsa...qcbhfo
qhilsgatefckbr..mjdn
iknmgltrcaejdq..hsbf
tsjghineflbcopda..mk
kjmnpodcabtsihfe..gl
jighlqfmncopraekdb..
rgqikhbadelopfjncm..

Type $5^2 3^1 2^3$
.....kmlsrqjpgiohfn
.....rqklmfinphsgoj
.....mrnpkshfijgolq
.....qlsmoifjkpnrhg
.....spmnlrqojgifkh
pmnrk.....dbacelsqo
nqpom.....crbsdkeal
qkosn.....beclmrdpa
rlmks.....eodqacbnp
korqp.....ncseldamb
jhsgqnapoc...dreibf
hiqpobceas...rfjngd
gshironbcp...fqajde
ipfmjabrqdgse..hlck
mgljhpscbqadr..fkei
lrihfckdenoagbs..jm
sfknledorajghmb..ic
ojgfileadbpnqhkmc..
fnjlgdoqkehpiacbm..

Appendix 3. $n = 5$ and $g_5 = 3$

Type $6^2 4^1 3^2$
......qopnsuvrgjtihkml
......unsovqhtirmpgjlk
......vmnrotugkhijlspq
......nvrpqstljugmkhoi
......tsvqnokjlghupirm
......spotrvjihqulmnkg
mnqrps......fuvdabtoce
tvrpqm......bdusonfcea
rupmvt......qabcedofsn
pomtnu......esfvdabrqc
vmnusq......rftepocdab
uqonmr......svctfeabdp
hsiqukedtafb....cvjlgr
skvlgircbead....jtuqhf
qltikharfbuc....vgdejs
itusrlbeqdca....khvgfj
nilvhjftucdpgeak...mbo
lghojvmbcutnakef...pid
gpjktocumvefibdl...anh
orsjlgpfdmbechqanki...
kjghinoqaspmdcrblfe...
jhkgopdaefmrlqsibcn...

Type $7^1 5^1 4^1 3^2$
.......suqprhjitmovnkl
.......mtpovuiqslnjkrh
.......uonmtkvrijlpqhs
.......tsvqoilkunmhrjp
.......rqtvushjkpimlon
.......nrsuplqtvhjkoim
.......vmunqjksrtpohli
rmvunos.....ctgqeadfpb
mptqvnr.....fuegadcsbo
punrsto.....acfegvbmdq
ovpnqsm.....drbfuctaeg
torsuvp.....qdabcengmf
klsvrutacbdf....ihgeqj
qtkhjrubvcsd....fgliae
hrqtkjlfdegb....vuicsa
sqiklhvdface....btujgr
nilptkjegobmvauh...dfc
uhjmoligbdtcefva...pnk
vkulminopfagtehj...bcd
jnhipqkcamesrgdlobf...
ljmoiphqngrabscdkfe...
isojhmqperfngblcdka...

Type $8^1 4^2 3^2$
........smrqltvijkunop
........ruopjkqtivlsmn
........tnumivsrpjklqo
........uomvsjtqlinpkr
........qrvtkliunpjosm
........ptsrqiujvmoknl
........nvpstrlkoumqij
........opqnrujsmtvilk
pusmvtno....fqdchebgra
tmvnoupq....cfbgedhras
msnptrou....hdavcfgebq
qtpvmnur....dseabofcgh
sqjlkirvgbeu....thafcd
ilrsuvqtfhba....kgdjec
ukqjrlvihsge....actbdf
rilkjqtsvdag....ubchfe
vpmunkilagtoechf...djb
nvtilokjcehduafb...mpg
onutpjmkbcdfvegl...ahi
kjiosplmdqfcabrhgne...
jroqimsneacbghkdflp...
lokrqsjpmfnhbgcedai...

Type $7^1 6^1 4^1 3^2$
.......wtrnqoimkluvjsph
.......vrtspumjlkqhwnio
.......qnurowlvthpjikms
.......spqoutkiwvjnlhrm
.......nvwtspuhmroiqjlk
.......tupwrvhkisnlomqj
.......ponvwstlhjimurkq
voqrtup......scewfdgabn
nuswpvq......cbrtaofedg
qsotupw......erdcvbngaf
pvwqnsr......gtcubfadoe
strnoqu......fwgacpvbed
uqtsrno......bevfwcdpga
jwkhvliufdabg....mectsr
kruvmtsgabefd....lwhcji
hjmuswlcevfar....dgkitb
rhjiwmvascgde....kublft
mkipljnoqedvawgub...fhc
wlvmkhjebfqgcduai...onp
ipnohkmfwguebvajd...qcl
lmhkirtdcobnqjsfgeap...
onpljikrgsctfadbehqm...
tiljqohbdapcnrfsmgke...

Type $8^1 5^1 4^1 3^2$
........tvspuilwmnkojrq
........vonurwjtsmqlpki
........psvqwukjiolnrmt
........swurpkiltvojmqn
........nuqovlrskwmitpj
........onrwtmvkulpqijs
........rqtvnjmulkiwosp
........utwnorsmvpjkqil
sqvtpwon.....eghcfudabr
voqrnust.....hdcwagpbfe
pnovqrus.....bweadfghtc
wunsovpq.....dtfrhecgab
rvposntu.....fcgbqwedha
ktmwuirlgahbs....jvfced
lmwjrtikcebfa....udvsgh
mkuivsjrwcetg....bhaldf
irsltjwvedahf....gbukcm
jwkulmviqpdchafbg...eno
oijmwqkpbhfacvude...nlg
upinjlmwabgeqchvd...fok
njrqkplmfgosdteihcab...
qltkmonjdrpgesbafich...
tslpikqohfcdbgarjenm...

Type $9^1 4^2 3^2$
.........opvnsujtlkwqmr
.........usrovjtwmlpnkq
.........nrpswvlmujotqk
.........rnovuwksjqlptm
.........qoutksrjwnvmpl
.........tusqjrmlvwkonp
.........pvturlwknmqjso
.........swqrlmvuopnkjt
.........vtwpmksrqujlon
tpowsnvqr....aeubchdgif
nupowqrvt....fbghdacsei
otvspwqrn....dgcibeufha
rvqpnstuw....cfdaioghbe
jwrutkslmbifg....evhacd
krwtvmujlchai....fdebgs
wjsvltmkufbeh....giardc
sltjuvwmkhfge....acidrb
qoukmlnpvwcdaehbg...ifj
vkmnjuowpaqidbchf...elg
umnlqpkojgebwidfv...cah
mnkqrjlsoiahcgtedpbf...
lsjrkopnqdgcbtiaehfm...
pqlmorjtsednfhaickgb...

Type $7^1 6^1 4^2 3^1$
.......trvouqixwjphmnkls
.......vpquwsjhikmnoxlrt
.......pvxnsouktrjlwiqhm
.......xtovquwrhsniplmjk
.......qnsprwvlmuoxhjtki
.......wupstnxilhqvkmroj
.......uwtrpxksjmlonviqh
oruvswt......fdaxgpqcenb
txwuonq......dvscbefagpr
wstqpxn......rfbgvaeocud
pqrxvso......twedafgbncu
npqwtru......scxvebdfoag
uwxtnov......baredqcgfsp
rtmiklsgfuxbv....wcjehda
jmkliuxbgcaer....hwvdstf
mvishtjregbda....xklwufc
vuljrmwscatfd....kgxhbie
khpnlviaxwgofcedb....jmq
invmqjhfabdxclgkw....peo
xjohwpkcdnqvegmfi....abl
lkjoximeqfwcphbva....dgn
sihpmklnorcagejutfdbq...
hlnruqposdegbmtcfijak...
qoskjhrdbefntauglcmip...

Type $7^2 4^1 3^2$
.......rsvwtxqikljmhnuop
.......xrqtpuvjwnshloikm
.......qpourwshnjmlvxkti
.......wqrsxtunmhlojvpik
.......supxvowmjktqirnhl
.......txsvwrokluhjmiqpn
.......vowquspltixnkmrjh
xoquptv.......bfsdwaegcr
otwqrsu.......gdxevcpafb
wvspurt.......xcgbaodfqe
qptsxur.......fgvwbecoda
txrowqp.......ebcvfgadsu
vrotqws.......cxagpbfeud
pwvxtoq.......uedcrfbsag
lmujixkdvtgbfa....cnwhes
nsmvjiwuafbgde....kxhclt
sulwnhxbfaecvg....idktmj
uikhsvjcteadgb....xwlmnf
mkprvjnagxdfqcwhei...lbo
rnikhplowbfqexdvma...jgc
kjxilmhfcgoepdvawn...brq
hqnlmkogduractsibfepj...
ihjnolmebcpsartufkdqg...
jlhmknipedcobfastugrq...

Type $8^1 5^1 4^2 3^1$
........ruxpnmvwiqlojstk
........qrptxlsjwvokmuni
........xvoqruitswpnkjlm
........osqwtvxlkmijnpur
........wtrvoilsupnmxqkj
........sxwuvjtmlnkiproq
........tpunqrjkvoxlwmis
........pqnsuxmrtkvwiljo
ptuwnsox.....bdeagcqvfrh
uworsntv.....gexfabpchqd
nvpsqrut.....ewcxfdhogba
vqrnopsw.....cuhgxabdtef
trxvpowu.....hfdbeqcansg
suvkxwlihberd....jmgfact
mskitujrcfvdb....hwxgeal
xklmvtrsagcfw....ijehbdu
rjstuxmkdwhea....cgvlifb
oxilmvpqenbahwkfj....dgc
jiqxwlvpnhgofdabc....kme
kowqljxmvefbgahid....cpn
wlmjiqknbdaxcfgve....ohp
qntujmilfoshpkcgrdeab...
lmjpkinogatcesruhbfdq...
ipnorkqjucdgstbamlhfe...

Type $8^1 6^1 4^1 3^2$
........tsvpoxlkmiwjrqnu
........oxqrsvwmlkpnituj
........rvwupqnsixomkjlt
........stxwrovnkjmqpuil
........wrtxqsmujnlpvoki
........uoptxriwvsjlnkqm
........quovtwjixmnklrsp
........xqsoupkltvirwmjn
qsvwxtou......hfgerdbapc
xtporsvw......ugedfbachq
oqupsrwx......dcbtvafegh
uvwrqoxp......btcgdhefas
vpsxtwuq......ehafgcdbor
roqtpusv......axdbcwghef
swmlnjkivgufct....axhdbe
nxkuwvjlchasbf....eimgtd
lutvminkhdcawe....xgjsfb
wkjsunltgfehdc....bvxima
klxjiqmobprehafvwc...ndg
pjnkvmirewfqgdxbha...lco
inlmoxqjdebcvhgafw...prk
trinlkpmfbhdagcjsuqeo...
jmrikptnacdgfbseulhoq...
mioqjlrspagbeutdnhkfc...

Type $9^1 5^1 4^1 3^2$

```
.........puowtkjsvqxlnmr
.........wqsvrtklxjnmoup
.........qvpuxlnwkrmojst
.........vtupqslxwmjrkon
.........oxrtwnukjlvpmqs
.........rovxsjtnmkqwupl
.........uwtrpxmvnokqslj
.........tpxsouvjlwrnqkm
.........srqouvwmtnlxpjk
prostuvqw.....eifhxacdgb
vtwoxrqpu.....fbcsdhgeia
wstvqporx.....haeubgfidc
qovwrxstp.....bghafdiceu
twursqpvo.....ixdfgeabch
xnmkuvlsjdicab....hweftg
kljxwsumneahig....cbvtfd
ujklntwxmfsihv....acdgbe
juntmkxwscgbed....vihlaf
ovxmpwjklahdbicfge...rnq
rkpnojmlvxdgqfwebc...ahi
mxqpvlnjkbfwdeacig...hro
nmsqlokurhbfcagdtiepj...
lprujmtnqgcefhdsabiok...
sqljknrotieagcmhudpfb...
```

Type $7^1 6^1 5^1 4^1 3^1$

```
.......ownvtuyljimxrqhkps
.......qrspowxilkynmhjuvt
.......xsutyvjmkwioqnprhl
.......rxqovyuhtlkpjwimsn
.......puywsqitvxlrhmnjok
.......wnxqutlsivjypkohmr
.......uqoxwptvhmsinrylkj
vtoxpnq......gyfuawdcesrb
rxwnspt......fcuyvqbadoge
pyrsxun......vwabtdeofqcg
suponrv......wgdfxeybcatq
noturqy......caesfbgxwvdp
xnyrqsp......efwtcaodgbuv
kmstwjuabvfex.....gcyldih
ywvklmhfdtubs.....ciexgja
wshyuvlefbdcg.....mxjktai
hkimvtxgycsde.....jwlafbu
ijuwhysvteaxd.....lkgbcfm
ohqpmxrygwbfnkjcae....ild
jilhyombprgqfakxdw....enc
lqxijkwcafynrdbmgh....peo
mlkjowidearpbhxycg....nqf
urjqilotvdnacseghbkfpm...
qvnlthkscpegomdbjufair...
tpmvkijnogcrabusedhlfq...
```

Type $7^2 4^2 3^1$

```
.......txourswvijlyhnkpmq
.......owrytpxmnuhjliqsvk
.......yrstvxoljhimwpnkqu
.......xutwsqrkhynliomjpv
.......souqywpimtkhnrxvlj
.......qyvrptuhwsjkxlonim
.......uvwsqrtjkmyxphionl
wrsoquv.......xcbtpedygaf
yoxusvp.......dgwfqabcrte
vtuxrpo.......segwcqyafdb
opwyvsq.......tudabfxregc
tvqwuys.......gdxerofpbca
psvqwty.......uxebargdcfo
qwptoxr.......abfveycgdus
luivtkjcebfxys....ndawmhg
kynixmuwsfgcav....dbjlteh
sntlyimvcexabf....gkwhujd
uxynklifadbgvc....wmejhst
xhmjirnefydboqwlcg....akp
mkjphowdqgafcbeylx....irn
njhrpwxgbqedfycakm....loi
jilmnhkpdxowegyfac....qbr
hqokljtrpavegdbsnuicmf...
rmkhjnlbtcpudafvisogqe...
ilrsmqhagpcouentvdfjkb...
```

Type $8^1 6^1 4^2 3^1$

```
........ovqwrxytilpmnjsuk
........tuxqsrnjkviypwlmo
........uwprvoxstkmjlyiqn
........vrsxtwkujmoiynplq
........wtrpuvlnsyjqoxkim
........rxusoywmlnkpiqvjt
........yowtqsjlnuxkrimpv
........qytoxpsimwnrkluvj
vtyrsowp......fxeahdgbqcu
poutrvqs......eyxbwfagchd
xuwsvpyq......achedbfotgr
wytpousr......vfghqcxdeab
txvowyru......bhdsfgqcaep
ypxvqruw......tgcdaohfbse
mkiwjxlvsfcgdu....ynbehta
svknywtxebhuca....gljmdfi
ismyujxthevbaf....lwckndg
jiluntvmdcfhye....bxwagks
qrnkilmyfpdawgcebx....joh
ojrxkmpibaeyhqgdwc....fnl
nqomxkilpgacedhwyj....rbf
lwpimqnjxhyefbdkag....orc
rnqjliokasbdgtmvufchep...
uljqpskngdovbciaftremh...
kmsltnjocqgfphubvieadr...
```

Type $8^1 7^1 4^1 3^2$

```
........rxtyuvpjmwikolnsq
........tyrsqpukxovilwjmn
........xusvpyrwnioqjkltm
........wtqurxyvoljpnimks
........svurtwqxlymnijkop
........prxqyuwmjvloksint
........uqywsrviktnxmopjl
........vpwtxqsoiukmynrlj
tvrswxpq.......ehbygaducf
wpqxtsyu.......hcfdbgreav
vwsqutxy.......bfecarpdhg
yspvqutr.......fdcwhbxage
uyxrsqwp.......dthgcebfva
rquwypsv.......gaxtecfbdh
ptvyxruw.......abgfsqhced
ojwmklvncfgdebh....yxatiu
nkiuojltygbahex....dwmvfc
kutjmynxhwagvcf....ldeobi
jnlovimkedcxgat....wfyhub
lxmnpkqibedcfhgywja...sro
sinljormfbhewgacydx...qpk
xmyilwojdspbafcnekh...gqr
iroknmjlqhepcdbtvaufsg...
mljtivkoacfhbsdugnerpq...
qokprnisgavfdtelumbjhc...
```

Type $9^1 5^1 4^2 3^1$

```
.........qsyuokmntrxlwjpv
.........tyswpnxmljoqrkvu
.........yupqxwjsvorkmtnl
.........opuswvkynxlrjqtm
.........rqxvstwkmypnlujo
.........xvryqutljnwopmsk
.........uwqotxsjylkpnvmr
.........vxopuyltkwmjqsrn
.........stwxrjnvupymkolq
txvowrpus.....eydhgcbfaqi
swprxqotv.....ahugfiydebc
vtqwusyro.....hbeadfcxpig
ypotsxuvr.....ciabhqwefgd
qyrupvsot.....iewxadghbcf
lknjvumwydietf....cbxaghs
nsmltwvyxhfbic....kjagdue
musvyktxlwgabd....enichfj
uvlknjxswbhdgi....mafycet
rlwxmyqjkcahebfgid....nop
pjxqolnkmebfrygchw....ida
wqynjoklpgeichmdxf....rab
xojyqpwmnirgaedfbc....lkh
onkmrtjqupdcfvlagsiehb...
kmtslnrpjfovhabuceqgdi...
jrupkmlnqactdgsvfibheo...
```

Type $9^1 6^1 4^1 3^2$

```
.........uptwqxjmovylksnr
.........psvrxynjmtwqoukl
.........xqrywtovnukmslpj
.........yrustvlnwxqopmjk
.........wxptyrvlukjsnoqm
.........vwyqrsuklmxpjnot
.........qtsvupyokwlxrjmn
.........tvqupwmyxjsnlkro
.........rywpsuxtjnokmvlq
urtysqwxv......cgehifbpda
pvurxwtsq......afhydigecb
qxsurvywp......ictagbfdeh
xywqtsrpu......bdieahcfgv
ysrvquptw......fxgdhceabi
stvxwpuqr......dhbfeyicag
jmowlyvukhicdab....nexgtf
vjxtkolmnibfage....cwyhud
nuymolxkjbhaeid....fgwtvc
owlnjxkytefbcvg....madihu
rpnjykmolcghxdfwbai...qse
wnjlpmqrxgadhfckeyo...bis
lkqsmnojyaexbcihwdg...rfp
tqmknjslodugfbaeivcprh...
mokpurnvsfdigehtaclbjq...
klpovtjnmsceihqgufbrda...
```

Type $7^2 5^1 4^1 3^1$

```
.......uywzxsoktjnhmqlprvi
.......ysoputrwlkvinxzhqmj
.......posurvzjhytmxknlwiq
.......tvzwpyxhinjkolmqsru
.......wxrvsutznlhjqiympok
.......ruyqzwvnxiklpohjmts
.......szxyqowikmlvrhpnujt
pquorxt.......vaeyfzsdgbcw
tprzvws.......fudbxcyqogae
oswtzpq.......xyafgdcbreuv
qxpswtv.......uezcyargfodb
zoxyqrp.......cdvwufgsbtea
yzqvpuw.......bctxesfoadgr
rtypsqu.......gvwdzbaexcfo
litmynzxwufeab.....kdjcvhg
uhiwxkmzavtgby.....lecdjnf
vwhjuzkctgbyfd.....emxialn
ivjutmhfdegbxa.....ynkzlwc
wumnkliveaxtgc.....jzfyhbd
hyoxnirabqcfzsejgmd....kpl
sjzkihygrcoadplbxen....fqm
kmnrljxbfpdoqeygcza....ish
xrlihyjoqfsdegmzbac....nkp
mlkhovnectawpqdfuibgjrs...
jnvlmsoqpdecruawfgthbik...
nksqjoldgbrvcftmhuwipae...
```

Type $7^2 6^1 3^2$

```
.......rvpwzoskylmjhtqxuni
.......vqwzosuxkmnilpyrjht
.......trvpyqzumjinkoxhslw
.......pzryutqwivkmxhlonjs
.......qpuvsxojnwzlmyhtirk
.......uxstwrvizhyknmjqopl
.......wsoxvytnuihzjrpmlkq
qrsuwoy.......vxzgfbceadtp
xpovztq.......ywdaucsrgfeb
rxvzypo.......gecwbfqadtsu
zspwoqt.......eayvgdxfbcur
tvqrusx.......zdbcwyfopage
swtorup.......dvxfyebzcgqa
uorptzs.......cgfxvaebyqwd
hnwlxiubfyaczd......jmekvg
yknxhlmfgebdua......zijwcv
wilmjkvyuaexdf......gnzhbc
luhyvxzawdcfge......nkibmj
mziknjwxdgueby......lcfvah
vyujiwhzcxgaeb......kdlmfn
jtxhsykoezqgapflnbci...rdm
kqysmnletbfrcxhjgdaz...pio
imztkhndyqsbprlcajxg...eof
njkipvrgafoqwcmbelhudts...
ohmqlrjcbtdpvgafkuewisn...
pljnqmisocrtfwbhuedvagk...
```

Type $8^1 6^1 5^1 4^1 3^1$

```
........xvotwqykuzlnipmjrs
........qytuzvnmkwjpxrolsi
........yopxrtmunvzikqswjl
........tqzsouxlmkyrpinvwj
........wrxzypjvimusonqtlk
........oxvquskiwymznljrtp
........vuqrpwlxtjiyzskmno
........rwyvqoitlxnkmjzspu
usqxvwrp......tdzghoyeabcf
rtxzqspv......wghdfecayoub
supwoytq......zbgfadhxrcev
yvrswpoz......cfbuxhegdaqt
pxuqrzyw......daetgcsobfvh
wqzupost......vychbfrdxega
jnmiyvzuagwbtd.....xlkchfe
zktvuxnydefwbh.....ajclimg
vmkjxnuiztbecg.....lfyhdaw
tylnkmwxgaehvc.....jbzfuid
mjwtikvlufayxz.....bghendc
lpyrmijkezhdsxbnfac....goq
xznmjliofpsadegcybk....qhr
nliosqxrbdgfayhzjce....pkm
qosyzrlmphcgfaejxid....kbn
owjknumscbdphraevltgqfi...
kivltjqnscrogfuhdewmabp...
iropltkjhsucebfwanvqdmg...
```

Type $8^1 7^1 4^2 3^1$

```
........pvzsyxtlwuormjkinq
........sxqyrwuvzmklpijotn
........uwvrzqptmxniyolsjk
........vpuxqtsyizmnlrowkj
........wrpvuyxmtljzonqkis
........tuspxrznljwmikyqvo
........rstuwvqxjkiynpzlom
........ytxqvzwknolprsijmu
swptxyqu.......cghebzfrdav
tyspzwrv.......uxbcfdaegqh
ztwuvqxp.......fhdysegacrb
wzqspxur.......gayvdcehfbt
rpxvsuyt.......wefzchqbadg
vszqtrwx.......bdafhgycpue
uqtrwpsy.......dvgbeazxhfc
nivkotljzbdwgcy....xfhmuea
iuojymtnhcezfba....kxdgvlw
ojlwnvzixgyebuc....akmfthd
jkyluzvmcegftah....obxdnwi
yljxqiosezbchgraknd....mpf
mxkzjonqdyahpfbiceg....rsl
xornmlkzaqfgesdjyih....bcp
qnmyljikbahdspfzocx....egr
prnoikmlgfwbdevhutajqcs...
kvimrspofhctadgebwuqjln...
lmuiknjwqdracheofvtgsbp...
```

Type $9^1 5^2 4^1 3^1$

```
.........pquyrvnlmwkozxtjs
.........zwoqykxvjlmpsrntu
.........qptxsjzkwurynmvol
.........rxyvwnkmtjozlspuq
.........tvzuqxmjlynkoprsw
.........sywoxuvznkpjrlmqt
.........xrpztwuyvmsqjokln
.........uoqrvylwxtznkjspm
.........otrsuzjnyvqlxkwmp
ursvzqopw.....atdghxfbyeic
vwqrtszxp.....ciubfydheoga
qvxtpowzs.....dyfhberaiucg
wouxrvpsy.....ebtfzacqhgdi
pyrostxuz.....hcbdegaiqfwv
lnwjumyvtfaxiz.....degcbhk
kujmyntlxvicwa.....fgezhbd
jxzlvynwubegti.....chmadkf
mtlkwzunjydich.....bxfgave
zktuxwvmlebdgf.....hiyncaj
xspzokjymgfhdbleiac....qnr
yjnslpqkoczfbdmaxeg....irh
nzmykjsqrdceapbghix....lfo
rlynmxkoqihspcgfazd....jeb
tqowjumrkasvhgidecnlbpf...
spkqnrltvwgaeofhcuijmdb...
omvpqlrjnhubfetwgkaiscd...
```

Type $9^1 6^1 4^2 3^1$

```
.........yxupsqjlvomzrkwnt
.........utvxzymjlnpkqsrow
.........qswvtuzxokyrpjmln
.........prytwxlzkuqmsnvjo
.........rvzyxpnmjwsloqtuk
.........sqpuyvxtwmkozlnrj
.........tyswpzovnjlxkruqm
.........vpxsurkwzynjmoqtl
.........xwrzqtvkulonypjms
quwvptrxz......dygeicfhsab
xyqtwusvr......caigbfezphd
vxspqrzuw......fgtaeyicbdh
zwxutvysp......icedrqbahgf
pruxzwvtq......giybhsdfcea
wzpyusxrv......tdhcgeabfiq
tvzmyljkuecahiw....dbnxofg
ojvwkmunxaztchd....fglyibe
ymjzontwkbgcadf....xihelvu
utlnmokyjhbgefc....zdxiawv
jqnsxylzogadrbiehmf....kcp
mprklzojyfdeqcahnbx....gsi
klyorjmqnzhfgesabxi....dpc
nsoqjxplmcibdghyfaz....ekr
sktjnpwmliuqfrebodvahcg...
rnmlvkqoswfhiabuectjpgd...
lokrsqnptdeibvgwufhcajm...
```

Type $8^1 7^1 5^1 4^1 3^1$

```
........wtqvpzumoyjnrslAikx
........zxyutswkmjvAqiopnlr
........sutyrxzAviljoqknpwm
........pqztuwrlyknxmAisojv
........yrpqAuvxnzomtlsjwik
........Azwsxvqylmkuinptroj
........xwrAvypujlmonzqktsi
........qsAwzpyjuxiklotrmvn
yAtqzupr.......awfxvbcdhsge
qrvstxAu.......wacfzheybgdp
sxpzrwyq.......vAuegafcdhtb
vzAuxpws.......cbghdetryqfa
xvyrpzst.......hgaweAdfqbcu
pqxwurty.......edvzbcgAafhs
utrxAqzv.......gcdyhspefabw
ijnyoAlwueaxgfc.....dkbzvmh
wokjminzcvfbeaA.....yhglxud
mloiwvuAeabhydx.....fjzgknc
Auzliyvjbhdgwea.....kmnocxf
lmwnkojxdAvfbch.....zyaiueg
kymAslipgbheqtdfznca....jro
jnitymkorcspdAgzehbf....laq
zpjmnkoifygcarsbhAdl....eqt
riqojtmkagczshbnfeAy....dpl
nwukvsqltpxdfgeoibacjrhm...
oksplnrmvfeahqtdxwuigbjc...
tslvqjxnhdurcbfikogwpame...
```

Type $8^1 7^1 6^1 3^2$

```
........wpAxqrukzjnvlystoim
........ysuwAprixnvlztmjkqo
........vywrztxoAkmnipluqsj
........xtsApuyvizljnqkowmr
........ruztxAqjoykwmiplsnv
........uwpysqzAloixjknmrvt
........sqvztypnwijmoAurlkx
........txrpyzsljvAkwmiquon
pvutxwsq.......yagecdzAbhrf
zytuprqv.......wfAxgcdhabes
xuAvszrp.......fcwyhbedgatq
rsqwvtuy.......cehazAgbfdxp
Azypqxws.......ghebdvrtcfau
upvyzqxr.......abcdAhsfetgw
wqszrupt.......edahyxbgAvfc
jlmxwnAkzfyhagv......ociedb
nijoymvxAegfhab......lzkcwd
imxlovkzaAdewbf......njygch
vkwnAymlgbfcdxa......jozihe
oxzAjkywcdevbhg......fanmli
mAlinozjfcagevw......hydxbk
snrmkjoAqzbacethydfig...pul
qtpklsimerhbudAzgfcoy...nja
yonqiAluhatdrscmkbzef...jpg
kristljnpgqufwdxvmobaceh...
twojupnidhxqvcebmlgfkars...
ljkrmitobvcsgfhdnxwaeuqp...
```

Type $8^2 4^2 3^1$

```
........tAyxuwsrkvnziqjmlop
........xustrqAwzmvojnlypik
........yvtAszrqlkjpmoinwux
........vrAwtsyuxzpmojnqkli
........sqruAvxznpijlyktowm
........wzxqvtusmlyAkioprnj
........uswrzxqyAjovnmtkipl
........Axvywuztpoklrsmiqjn
usrywzvA........edfbgaqhxct
Aqyvsutz........fcxwdebghra
wtqryAsx........uehdaczfgvb
rvAxqwzy........gfdhctabseu
suvqAyxt........agcfzhrdebw
vywzxtqu........chegbfsAadr
twusvrAq........yxbehzcadfg
xrsuzqyw........vAgatdfebhc
nliAujkmfwabcghd....ypezvxo
omxkinupbyzvdhwa....elAcjgf
pinwmxjozfchbeav....Agylukd
lxoikmwndbuzfyeh....pAgjcav
yozjlprkghecqdtfbaAi....nms
zAplnsoiqdfeyrcgjbak....mth
kjlmtopsrcgdhabAinzy....fqe
mzjnpkirhgdaeAfboylc....tsq
ipktolmjaeqfgcvxhuwnsbdr...
jnmprilvctbsafgedwuxqkho...
qktojvnleahgxbdcwimufrps...
```

Type $9^1 6^1 5^1 4^1 3^1$

```
.........wypsutxjvznroAmkql
.........uvrAtqwzkmxosynjlp
.........rwztAvkxyjusqplonm
.........yxvpwuoAjkzlnmrtsq
.........vztxqsAknwlpmjyuor
.........qtyvszjwuomkplAxrn
.........zAwuxplyovjnkrqmts
.........pquzrAvlmyktjosnxw
.........xpswvrymlnozAtjqku
zwvArytqs......afxudcgbphie
yAwpsvrux......gafibhtqzecd
uxpqtsyrA......ibzhvefdgcwa
rvxuqpstz......biceAydfawhg
psqxywuzr......ecAdhfiatbgv
sqAtwzxyp......hvegidrcfaub
xuzlmovAwiacfyd.....jenhgbk
Ajuyvnzwogfdbac.....mhkilex
vlowxjkmneuagbh.....Ayzcidf
wnmzkljxuAgeyif.....abhodvc
nyjmuAwlkfcbhdx.....gzievao
lzyontqkjcsArgamhdbe....pfi
otkjArlnmdeiqzycgbaf....sph
tolkzqpsyhbfecgndiAa....rmj
qmnsokAplbrgdhizeacy....fjt
mrsvlunjtadqipwfohxgbcek...
jktrpxmovsihaebunwfcqlgd...
kprnjmovqthxcfeduglwiasb...
```

Type $8^1 7^1 6^1 4^1 3^1$

```
........zxwvyrplmBAjnkquitso
........trsuAwqkoivzjpBlnxmy
........vyqwpxszljBmAuortkni
........pBrqvswAznjymiltoukx
........wqvzspyiBolkxtrmAnju
........svyBtzrnxkmowqAjupil
........xtusBqvjAyoizmpnrlwk
........BztxuyAwjvknolsiprqm
yqBrwtsx.......cvgdAahezbfup
spABxurq.......gbzhwycdafevt
tvuzqAxs.......ywhbdegfcBapr
purytvBw.......bgxcafAzesdhq
rxyAuBpt.......hcewfvsaqzbdg
Arqtvwuz.......fadxbBegphycs
BtvqzsAp.......xfweghrubacyd
mozjynvkedfgwhB......acAlixb
wBxvikolyhzcbAf......nmdjgae
xilmkyzndaAfcbe......Bjhgvow
jznxmlwAabeyfcd......oiBkhgv
vAjlBiyogwxezah......bnkdmfc
ljkonmiyAeBdxva......zhgcwbf
zminjrqBcApbhguodlfek....sta
unoksptmbcaArdzBefghi....qlj
qlsuozmifpchgtbekAaBd....jrn
kstiAqjurfbpdBcmnazlg....oeh
iypwrokjugdaqetvhcnxlfbsm...
nkwspxlvquhrafgdymicbjtoe...
owmpljnrhsgteuxaibyvcdkfq...
```

Type $8^1 7^2 3^2$

```
........zvrpxyqBnAowmkjitlsu
........tzvysAuxkomjBirlpwqn
........rAqvptyzwjnBklosuxmi
........ysuwvBtAlnxmizkrqpjo
........xuArtpwizlkyoBsnjqvm
........prBszuxwjklAynmovitq
........vwytBszkmxjiAonqlurp
........BptquvroAmwnxyizsjlk
pBuqtsxw.......cbfAdaezhgryv
rtApsywq.......aBezfchuvbgxd
yxBrqvsu.......bgzcawAdfetph
tzpvAqrs.......edagcfxbBhyuw
xrsyvzpt.......gfBhedwquAbca
vAqswBur.......hxgyzbaptcedf
usvtxwqB.......yhcdgzfAprabe
lwyxznmicBahbed.......gAokfj
kolnmjiywhxBdgA.......ebzfac
nlizjAykgbcewdf.......Bmahox
jyzwnmBAbcdxghe.......akfoil
BijlkxzoAdfbewh.......camngy
mkniyoAzhxwfcag.......ljBdeb
imxBolnjeyzAhba.......fdkcwg
ApkjitonugezqfvmadBblc...shr
zutorpjvfebdAqBncialhg...mks
sqmABrlpdfguazcjohikeb...vnt
qnrmukvlstpayxbdewfogjhci...
ovwupikmqahcfrslybexjdtgn...
wjoklutxaqsgrcpfiybhnmved...
```

Type $8^2 5^1 4^1 3^1$

```
........ButzryxvjwipmqskAlno
........qvywsABzkxnjourlmipt
........zxrBwquAnjlvyiotksmp
........yzusAwvBiokmltqjrpxn
........sBvuqtwxAlmkzpnrijoy
........ArstvxzqoywBklpnjuim
........vqwyxurspkBniozAtmlj
........xAqvyBsrwpoinztmukjl
qBuzystv........eaAfwhgdcxbr
rxtvAywB........bcfgesuqzdha
xtqyswBr........zbvhAcfegadu
zvsurxAw........ahcygebBqtfd
yqzABust........vdawxrhbfcge
BsyqvtuA........hfdxagezbrwc
vzwtxrys........BebchaAudfqg
swvBuqry........dzhAcbafegtx
okBxpjzlgwfdheAc.....mianyvb
piAjmknzfaxedgby.....Blhowcv
lyjwonvmbdgxzfch.....ABipeak
nAipkvxoefBgazdb.....jmclhyw
wmnlzApxhyefcbga.....djoBvki
kjloqBiprcAhtsadgmzeb....nuf
AnkmlijqtgzaurfecBpod....bsh
jprsnzlkdhbABctumigaf....oeq
iupktmonceabfdhglAjzB....qrs
mronipkuatcqbvewxgyljfdsh...
toxiwlmjubhrgaqfynedvkcps...
ulmrjoqiwsdcehytfvxbpnkga...
```

Type $8^2 6^1 3^2$

```
........xqzvuAtroBpykisnjlmw
........wvrxzsyqBlmpnkuAiojt
........usqwAyrBjxoizlpvnktm
........zBwsrqxAyjlniptkomuv
........quyBtxzwmAkljnvosirp
........rwsuxtBzpmjolAkqvyni
........BrvAszwukoixpmnlqtyj
........txAqyruvlkBwmoipzjsn
txuvqAsz........hdceBabrgwfy
uztBsrqw........xayfAecdbhvg
stxuByvq........efwAhzgacrdb
vwBqAxyr........bhfzcdautges
Avqrwszu........cexByfhtdbga
qursztwA........gyabfxBhevcd
yrszvBut........fwAcdgebaqxh
rqvxtuAy........dgzhwbfsBeac
ByAnimkocegdbwfh......zjpalx
znjwxiopeyafBgAb......mcldhk
wlyomkinazhbcBed......jgAxpf
lAzjnwBxybdhgacf......omkpie
ijplynxkbaBgdfhe......Azmcwo
omwikzpjAgxehcay......dBfnbl
jikplvmBsAerqhdtzbnagc...fou
mBnAuplihdftvegcazbkoj...sqr
npomrltsfhczevbgAidjaB...ukq
pkiyjorlvftawdsxncgmbhqeu...
koltpqjmdcuyabvsinegxwrfh...
xsmkojnvgtbcfuqawphdeylir...
```

Type $9^1 7^1 5^1 4^1 3^1$

```
.........xvtqruBnAlyzsjmkwop
.........sqwtuvzkpxmoArjBlny
.........AzuxBrwokyvptqsjmln
.........rsAzwBymovxjupntkql
.........wtxuyArvlknmoBzqpjs
.........zuqrvxAwmBoypktlnsj
.........uwvAtqspznlBjokrymx
.........yrBvAsxzjoknmlupqtw
.........vAzsqytjwpBklnouxrm
xuvBysrtA.......igbfdqeczawh
tsrqBuwvz.......hyiAxfdbaegc
ywqtrxusv.......BazehdAgbfci
qvtzwryAs.......bBgaehfidcxu
wzyAuvsrx.......fidcgBtaehbq
rysxqAzuw.......dfebachBitvg
uAxrsyBqt.......ahwzigcdfveb
mjzwxonklfBayhce.....igpAbdv
jBwkAzvpmaxfgdhc.....eblniyo
BoAnlmxwpbycezdv.....kihgjaf
vxojkwplygfBcah......nzAmdie
ApBlvnjxkcbywifd.....zmeogha
ztlumjonBqhdfseagbAic....rpk
nkmspBqzutegcabiAdhjl....ofr
lmkpjtABohirbezqcnagf....sud
sqnozpmjrBdeabiglcfhA....ukt
kluvnqtojighdxwfyempbasrc...
orpmtlkynecihfguxvjdwbaqs...
pnjyoklmqdasigtbexcwvrufh...
```

Type $9^1 7^1 6^1 3^2$

```
.........wxsqyAvolzkBjptrumn
.........ztrBvywlpnAkxomusqj
.........qwAyuxrmBpnzotvlkjs
.........xuBtrzAyowmlnjspvkq
.........uqxzBtsAwljypromnvk
.........tAwusBqxyjoplkrzmnv
.........svyAwuxjmkzoBqlnrpt
.........rBzstquknyxmAljvowp
.........BzuvqrtnjxlwymkApso
vzAuxyrqs.......Becdibhgtafw
utqysvAxB.......icfeawzhgdrb
zrBvusywt.......afAbhgiqecxd
wvurztqAx.......eagydcsBfbhi
qAtBrwvsy.......zbhgxaduceif
ywxtvusrA.......diBfezcbqhga
tusqyrBzv.......wxeighAadfbc
myzwnlpkjcihfdbB......aAoxeg
jxkpBomynAbgwicf......ezaldh
ljnokxzBpgcbhAea......fdiwym
pByAojwmkafcezdb......gnhilx
Ampzlknjwdgfaxhc......biByoe
okjlAnxpmhadifwz......Bebgcy
nsvmqBjuzfebhgidcopAk....tal
sqmkpAonlvharcfegzdBbi...jtu
BplnjzktoievgbsdfAahcm...qur
klrjwqtouydixeagbhmcnfvps...
rnoxmplvqestcaiyhgbwjdufk...
xowstmulrbyqdgvhpkiafencj...
```

Type $8^2 6^1 4^1 3^1$

```
........vuwBxqCsmkylzinprAojt
........qBtuyxACwizonpjsvrmlk
........BCrtqAzykowpinumjvxsl
........szuqCyxtjpBAwovliknmr
........ytBwzuvAljmikxrCqpsno
........ArvCBwsuxmlkpyijotzqn
........zvsyrtqwpAkmClouBnixj
........tqzvwCrBoxjnAmskluypi
tzByrvAw........hebcxCaqgfuds
BCywtAxs........adfgczeruqhvb
vtsCuxrB........fzahywgeAcqbd
sBCrzwut........Aghxabfvcdeyq
qszuxtvr........ChgBbeAfdacwy
ruwqvszy........cfACdgbthBaex
AyusCqwx........dBczgfthabvre
uqxzAyBv........gaCwedhbtsrfc
xlpiBjnCgyazAdhb......comefkw
ypnAkijlacxebfwz......dBChgom
jnkBloizfAyagbdx......mceCwhp
pwAoimCjxgefdayh......Bnklbcz
kjoxwzymdbChceBa......pAnglif
ixlmyponbwAchBed......Cgfjkza
niqkoutACecrsvafblpjmB....dgh
oAmvqBlicsbdfhurnCeajk....ptg
CmijnksphdqbargeBcoflA....tuv
lotpmCqkrfgAvsbcindeBh....jau
mrvnjlpowahguztqeyxbfckdsi...
zkrtpnmquxfsegcvywidhjlabo...
wvjlsrkuehdxtcfgzbnyoaqipm...
```

Type $8^2 7^1 3^2$

```
........tuvryBwqipnmClxAojkzs
........yBwtAxCzmnpkjilrsuvoq
........rCBvtqyAlxinpmzwksujo
........zrsuqytBnkjpixAoCwlvm
........qArBsvutjoClynmpikxwz
........BztqrusxymoilACjnpwkv
........sqzwxCrvkjmBAyoiptnul
........wvuzBsqyoAkjxpiCtlmnr
uytAwsvx........afBzeChbgrcqd
zsvqruxw........CgAedcyBahftb
AwBsCtyu........xazgbhcvfqdre
trwCuBsq........Ayexzfdcvgbha
rvCuqxwy........dzhbaBegAcsft
yzxwArtv........gBbcfeasuCqdh
wqAtvyuC........bexhBgfardzsc
Cuqysvrz........fcdAgaBehbtxw
nopmlCzBahgbfcAd......keijyx
xlyBkiopcefadzgC......mjAhbn
knozplAjxbecCgdf......hmByai
pjkoBzCmAaxdhecg......nbfily
BpixyokAdgChefzb......lcnamj
oizjxnlkbyACcafh......dBmegp
mCjliABnhcygzbxa......fdeopk
jAsvmwniutqfadBreblChok...pcg
smuntqjlCwaAvrheBdcokbp...gif
lBminkqtgdhswAvcpCfaojb...reu
vxlkzpmrfscebtawhigyndjuqo...
itnrjmpovfbxuweszhydckgqla...
qkrpojisexdyghbuclafmzntwv...
```

Type $9^1 7^1 6^1 4^1 3^1$	Type $9^1 7^2 3^2$	Type $9^1 8^1 5^1 4^1 3^1$	Type $9^1 8^1 6^1 3^2$
.........ByszrvwkoCmxApjuqtln	.........uqxvtyzBAClmnokwpsrj	.........BsvzwxCAmqypluktrjon	.........xuzrsACymljqBkpovtwn
.........uzrsCwvyAmknBjqltopx	.........xBAwszunlyojmCrqtvpk	.........syACzrxvomlBjtpknuqw	.........AytvzuwCjnpBxosrlkmq
.........twBCszumjkyAxorpnlqv	.........rsuqwBAynlkCzxtmojvp	.........wztBuCrxpAjlqvsnmoky	.........yvwxuzsBnoqkmCAjtrlp
.........qACwButzmxnlpvkrsyjo	.........qtCsAvwxmnBypzlrkoju	.........yAxvCuzrkjqnwlBmotsp	.........BwusvytzlmkCAxopqjnr
.........stAvyxClwnokjrmBupzq	.........zrByqutlkojACpnsvmxw	.........AwyurstzCkojmpqBlnvx	.........rzACtwxvoplynqjkBmus
.........yBvAxCsnlpzjwqtomkru	.........ywsxuAvCzmnlBkptjrqo	.........vxCtBwuynpkAojrqszlm	.........vxBtyrAwzCmnkjqsupol
.........xquywrzpnoBCmlAsjvkt	.........CxquvtrjypmBAnolswkz	.........zuBrAtywqonCkmlvpsxj	.........tsCAxvurBzypjlwmonqk
.........vxwqtsrABjCmzkonpuyl	.........szvtxwCopkAnjBmurqly	.........uvwytzsCjnBxAomlkrpq	.........wtryBCzukjAoqpnlmsxv
.........wuytzBxjkApolmCqvrns	.........wAtzCrqmoBykxlvjupns	.........rCusyABtxwmqnkopjlzv	.........uBvwAtrsxqClzyknjopm
CyAtwuxrv.......fgidBbeachqsz	urvwyqAxB.......fCghzabdciest	CvxwtBsur........cyAfdeabghiz	BstCvzury........Acxiehfbagdw
vtwqArBuC.......xedgzycsfihab	sBAzxtvwu.......dhecfiyCaqgbr	srutBvxwy........dbzafACceihg	rxCBsAytw........cieabzvhfdgu
tByuCwAqx.......hizcagfbedsvr	vyzruswtq.......hcdxigAeBCafb	vsAzwtCrx........fBigchuebdya	urxtAvwsC........abchfBgidzey
sutBrzwCq.......gycxiahvAbdfe	zuCsqvBrw.......Agxicbdheftya	zwrAsuBCt........yxhegfbadvci	wvuArxtCs........iBfdcghabyze
qsBrtAuyz.......chbawedfvCxig	BAwqryxsv.......zfibgheaCdutc	AuwxzsvyB........hcadCiegfbrt	vwrxutABz........dahcyibgCesf
ACuzqvsxy.......iaefbhBdtrwgc	CsruwBqAt.......eihgxyfcdabzv	xBsuCryvw........Azgibcdfteah	CtAyxwsvu........gdbzafeBrich
rvzsxyqtA.......BCfhdiguaebcw	wqtxvryuz.......gbAChcasiBfde	rzyBAwtsC........adehxbiuvfgc	zuvstBCyr........exgbiAdwhcfa
kAmnyjolweahgcbi......CBdfzxp	knyjozCpmAebaBid.......fhglcx	utvsxywBA........gfbziahrCced	tBwzyuxAv........hfigCecdsarb
BzxClkpwogfabiyc......nejAmdh	nxlCpojBAdygfiac.......bkezmh	wxqmlzpAohBdibyfg.....Ccjakne	xnzomjplAacyBfghb......iCeqkd
plnAjoyBmzCgxdea......bihcfwk	jConBxmkpabzchde.......Aflyig	pkCyonmjlxhzdacei.....qgABwbf	nkylCpzoBcidabfex......mqAhjg
wnlxoCjmpAdzafcB......ihgkeby	ApxBknzolifcCyba.......jghdem	ylojpCzmnbcfexgid.....BAhqawk	AmkjlqnzpCbfdcByg......aeixho
ypowmxzklfcbihAd......agCBnej	yopAljkzCbafgeci.......Bnmxhd	lmjkqxozpaeiwdBhc.....gnCAyfb	ojpmBkqxniAehCdfa......lcgbyz
owjpnBCzkdexfgiy......Acblahm	mzkpClnjxBcaedfy.......gAbhoi	ByzCnkApqegabhiwf.....djocxml	mlBpoyjqkdaizhxge......CAnfbc
nqrjpmtosChdBvgbaclAek....iuf	lmBkzCpyjhdebcgx.......ioAnaf	qjtlrAknvdasfchgbeiCmB....puo	jymnzCopqheacgbdA......Bfklix
jkCvulnsBhrecqtAbpaifo....gmd	ptuojAsCnfvhBregiacdblm...kwq	tnlqvmjoucihaseABbCfkp....gdr	qAoukrljtfCgeasBcbhdmpn...wvi
ljsmvnkAuagtreqhdfBbpC....coi	qlsvAwtmkghirbCBadfepoj...cun	jABnuplqkCrbgfdvhiacoe....mts	pqnrwlmkosgbfeivhCABjdc...uat
mopkBqlvnrsiuAhfCbgecd....jta	owntmkuqrvCdAfshbBjpaec...igl	nCpomlrkstfgAivbeBhdca....qju	kCswpnBulefhbrcidqgaAom...vtj
xrvlstmpjibqdafeozywhcunkg...	tkjmnpolyeurizhscxbadfgqvw...	kompjquxzftrheacslgwbynvdi...	yzjkqsrnmghciwabtfeoxlduvp...
zmqokprntbvceudgwxhjyfslia...	rjqytmlvscgwhaxbkezfodiupn...	opnvkjqtmidecgbauwlxyzrfsh...	splqnovmjbdxuihcfykzegartw...
uxkyzsvjrcifhbaqedwlgntpmo...	xvmlsurnotiydgqfpjazekhwbc...	mqkryonljgbcxvfdazepwhstiu...	loqvjmkwxzrsgdeaipynfhbtuc...

Type $8^1 7^1 6^2 3^1$	Type $8^2 7^1 4^1 3^1$	Type $9^1 7^2 4^1 3^1$	Type $9^1 8^1 6^1 4^1 3^1$
........prAswCtkiyvlnDoBujmxqz	........vstACrBqipozylnDukwmjx	.........ABvyrzCkplnDxmtqwuojs	.........usBrwzytqClpDonjmkvxA
........xtCwpzBjyvloirDqkmsAnu	........tuxsBqAykDnmjizprColvw	.........srwvuCDmnoBxzjqtkpylA	.........BvtDusCzlkqjxnomrpAwy
........tyxuDqwmCjknApsrioBvzl	........BzvqrCxwDmykAnpuitljos	.........DqCBytrjAxpnkzowlvsum	.........xyrBvwuCAqDmnptskoljz
........rAuyxwDolkzBCqmtpsnijv	........rADCqxutzkBnpyoivwmslj	.........CtyAwuzBDmxlpnjvrkqso	.........rwuzDtBxolkACyqpvjmns
........BpvqtAsDkzxmwnrjCliuoy	........zBsyxwtAjlmoiDCvprqkun	.........rvtxCsADmjypokuBqnwzl	.........wAvCxrsBzpoyqkmDnljut
........AxrtsBzwmoyiDknplCujvq	........yquzvtDrnxplBomwCsjAik	.........qzswxBvlyDjCAoruntmkp	.........zrCtyxvDBnmokAjluqpsw
........wBtvyDCxoljAzskmqnprui	........wtzvyDCBAjkinmlospruxq	.........wsDuzqxClnkyBpmrjovAt	.........tBwysvAuCDpzljkoqmnrx
........vwzDurqAxnikyCjosBlptm	........AyrBsuwxoizjmpDClqnvkt	.........zyAsBrqxjCmknlwpoDutv	.........CxAwtyrvDozkpmBulnsqj
sqCrzxpu.......yegAwvBdfacDtbh	yuvzBqtC........xahDfbdrceswAg	.........uwrtDxBzkyojlAsmpCnvq	.........yCDurAtwmjnxBzpksvqlo
vyrzxsqA.......ChwbfBepudDtacg	wvCxuyrD........hBeAzagsdbtcqf	DqvuCwszr.......fiAagexcbtBdhy	zvuxrBACt........bayihDsdgwefc
AzuxvwrB.......cgDCaebqdthfyps	AwqBvtsz........yhCbDgxdefcaru	CArvtsqyu.......gBfzcDeidawbxh	CwrztvxDy........gAdhbcuBfsaie
wszturyD.......vABdgxahCbpeqfc	qswtDxuy........BfACchegadvrbz	BrzsDuCqx.......Aehdigbvcfatyw	DByrxzstu........efhgAawiCcbvd
qrAvCpzw.......hDeaybuBcftgsxd	vDrqxzwt........CAfdechBgauysb	AvtzurwDq.......dCecayfhiBsgbx	BuvwsDCyA........dhxezbirtfcga
DABpsyxt.......gvdfChcberuqzwa	ztsvrwqu........dbxyCABfDhagce	uBwtqAvxy.......hzigecCbsDradf	rszBAyuwD........cbgCixefdathv
tuvsqABp.......fzCcxahegDdrwyb	Drzwsuxq........gCaebdActvBfhy	stqCwzAuv.......yxcebaDBfgdhri	vrBtuswxC........fyADadcheizbg
mByADnwxdCfbzvh......ogaciklej	uBxszCvA........edgcafyqwDbhtr	quArzvxtw.......acBChiygDsbfed	sDAuvtrzw........yibBefCgcdxah
lixjwBokzhyfbgv......dcDeaCnmA	BpylmjokxadbcfzC.......nhgDieA	mkClpozBnyAhdfDi.......eabgxcj	tCsAzxBrv........icafyghbDewdu
jwDyAmkoevbzchx......gClBfadin	kylnAiCxcbgafBhD.......jmoedzp	lzoBxyjpmeibDghc.......kCdfAan	lzxCnjyAoeDfcBhdi......aqpgkmb
yloBiDAzCgwcvae......mfhnbjkdx	mApDolzjdxbghafe.......kBcCnyi	pCBylnokzxdfgaAb.......Djihcme	pqklynomxacgACdDh......bejBfzi
BmjwnzCvycDdAxa......likhebogf	xknyiomlCDAdzbaf.......ejBhpgc	oxlABkDmCdfgbciy.......nhejzpa	mjqnBolpzbfdxAica......gChDyek
zxikoCvnfDdaBey......jlbmgchAw	nxmAykBiafChDecb.......lojgzpd	klxpABynDabdhifg.......Cecmjoz	omCkjqDlndexbzgaA......fcBhiyp
pCmnBtsqDecgdfuljiohk......bar	lCojnAyBDcfxgzbd.......akipemh	ypjxnDmAlBhzabge.......doCcifk	yxljqpmBkgzhdbfec......DaiCoAn
nokCmquihsgrftcadbBDj......elp	izjmCnApedBDagyc.......hblkxfo	jnmDyxBCofeazAcd.......lkhipgb	xAjpoCzkqDiyheBfg......lnabdcm
ojpqriDsauBeCcgdnmhbl......fkt	snkpqrivhCwftcgubeDalBj....odm	nmpqstljkcCBievfbgaDodh....rwu	uopqDlnvjstbeiawdhgCcmB....rkf
CDnotlijqbspaurBfcged......mhk	tjBCkmDnfvqudhrsloipgec....bwa	woumvCplBtDieqaschdbfjg....knr	knDvmwjqrchisfubepdBaoC....gtl
untilkjmsdqChpbeBaDcf......gro	CqDupBjmswcredvhfnbgoki....tal	twDnkmuvjigersdhpobfBCa....lqc	ntwDkupjBvgsfdCibameqcl....hor
ktlDpumCgqaBrdfibhnjo......cse	jmirtDlobhewusdapgcBkCf....qnv	vDnwrjtsphcqCdbuoaklmfB....eig	wlnsCktomfacvgDhrjBibde....upq
ipqlyonrufexgbdzwAmvctasjkh...	roAilspwqgytbvezacjfhxkmnud...	xysjolkrtbaufhewndziAmcpgvq...	jktywmqspAueahbzfnxclgidvor...
rksmjvtlbahAeypncxwzgfuioqd...	olukwpnsgraeAyqvczdhxjbtfmi...	zjykmprosvxcqtwaebgAdhifnul...	Apmolrkushdziacgyxejnfqvwbt...
xvwukjlyczphqsAbafedmitngro...	pitojvkruehcwAsgmylxdzabqnf...	rskojqnwAguxcvytifphzbdalme...	qyompAvnlibagcexskzfdjhrtwu...

Type $9^1 8^1 7^1 3^2$

```
.........uvzDyrBAnlCokqjmtwpsx
.........vxCArwtszpBknmljDuyqo
.........CDwBtsuxAmjnqzkpvlory
.........tuDrzvAwonkqyjmCBsxpl
.........wsrtByCzpDlmonAuxjqkv
.........stBvxDzyjkpAlCnqomwur
.........xCtsAuvBDyzljpqrwokmn
.........DwyuCArtlqnzpBoskxmvj
.........BrAyDtxvCzqjmopnlkuws
sutwvrDAC........Bhgifycaedbxz
vArBzyxuw........fahgCedbsDtic
xtAuywzDs........iCdfBghvcraeb
BxsDruACz........ceybhafwdtvgi
wyzxsvutD........gBacdACirefbh
CBDswxrvA........yiehzdafubgct
rsuAxzvyB........ediCaDbcgfhtw
yCxvutwzr........dgAaDbBeicshf
AzynmCqopbBeahifD.......ljgcdk
DkBonqmjlcgifeayb.......hpCAzd
jlCmBknpoaAchfgDe.......dbizyq
mpozkABqjycfibCad.......Dnhelg
kDnyojplmizhecbgC.......BfqdaA
zjqClDymkAedbaBcg.......ohnifp
qmjlDpokyhbazdcif.......gCBnAe
pqwktlCBvrdsxufehmboDgci...jna
lnmqCBksteivdwxhabjDpcfg...rou
orvpqstnxdfuCghwiacbBekD...ljm
uwkjpolrnfyxgizscqAmdbhetav...
tvprAnjwqghbcsedukofyilzxma...
noltjmsxuzagwvdbrhfceAiykqp...
```

Type $9^1 7^1 6^2 3^1$

```
.........rBxuDwzCjlnymoEvptqksA
.........uytDvABxCzomEpjklrsnqw
.........vqEArswpByzoCnlDjmtuxk
.........DAuByCvzpwjxkEmlrqotns
.........qsrtCyDAEmlwxunokvpzjB
.........zvqyBrElADpjwstnuCkmox
.........xEzwsBqDynAkjmCtvorlpu
.........EDsCAqyozBmlprkutjvxwn
.........tCwvqxAylEDzBksjopnrum
ArBtzvuEs.......egxdfiqabCDhwcy
uCrztsAvE.......cwdxDgbqhefiBya
xzAvEDtwq.......hcgyCadrfsuebBi
zxtuBqsCv.......wicgAyhbadEDerf
qvzxuryAw.......EbCaBDthgfsdiec
DyursxwzC.......dehBiAfvqEgbcat
wtCsAyDqx.......BfiEbhcergaudzv
pExjwCByzAhcbdia......Dgemnfokl
ypjkoBlmDdwagxzc......ifEneChAb
noyAxwjDlBadcEeb......Ciphkmfgz
klEpmzCnBweyxfgh......jocDiaAbd
oBwDpjnxkCbAfzae......gdmclEyih
BjlynkzoAbxehwEf......aDCidcpmg
vDoEjlkptgrfqbuimnaCcd......she
snkoDEqlrafhiudCgmecpb......vtj
EkDnCpoumetbdhcsjafigl......qvr
muplvtEjoicCsaDrkdbehn......gfq
tmvCqnrsjhgDEcfuiokbde......alp
CsnqlmvkufigrebtaDphEc......jdo
jwqByomrpsuvzgtdnxAkafecibhl...
rAmwkuxBnydiatvgbhofezlpsqcj...
lqsmrAptyczBeihxfkjwnovudabg...
```

Type $9^1 8^1 7^1 4^1 3^1$

```
.........CsutvrBxnmyAzEDlqjpwok
.........wBvsADrzjqCoEpnmxuktly
.........zvwBDtuyojmCAlEqnrxskp
.........vCtxrzyDkEjmBAqusowpnl
.........rxAuwBsCEokplqyDvntzmj
.........EAryCvxsqznDmkBwplojut
.........ADBzEsCuynoqpjlvtwmkxr
.........xzsABuEwlCpknymtjDroqv
.........suxrtwvEAyzBqDCnkmjlpo
rDvxytEBu........gaihbCfcdesAwz
vwryszDut........aBfiebcxEdCghA
uxzrwAvEB........eicgDhbdCsaytf
zsDvCEtyx........BfAciegbwahdru
sAwBtxuvD........zcEeydhgfCirba
CtuDxswAy........hebEazdrigcvfB
DvxsECAtw........dhgycBearbufzi
ECstuyrwA........DbhzdgafcveBix
poCjAkznlaygidbfc.......EeqDmBh
yzmknqplCBiDhgEae.......jofbcAd
qEAmzDnjobaCfyihB.......pgkledc
mpknBolzEyfcahCbA.......eDidqjg
AmBpDjyonghzCfacd.......klEqieb
oynzjlBqkDbEcaAdi.......Cmpfhge
nqpAlBkCjicfDedzg.......obhEaym
jktoquCDscEawxfgrpledhmi....bvn
kulCrvqpmegiEbxthcdDaofj....nsw
wjEqkromvtdbeuhiaCDlfgnp....xcs
lrjEvwmxphtebsgDfikdnCco....uaq
tBolpmxsruwydzeAvfgqbjikhacn...
xnyuopjrqfedvicwbmABlkazshtg...
BlqwmnskzdrhgcyetbpajfoAiuxv...
```

Type $9^1 8^2 3^2$

```
.........sBvErADxjlCqknozwputmy
.........uzEtyDvrCBnmlqpAxjsowk
.........vwtAxErDBCjpzmlnosqkyu
.........tyDzAwsvkmEjolqCuxnprB
.........BuyvECtwnqkAjpzDroxlsm
.........CEwyutBslkonpzDqvrmxjA
.........rABuCyxtEnplDomkqvjwzs
.........ztrCvsEuqDlBAkjonypmxw
.........wCxszruBDAmonjEptkyqvl
wsEtBrvuA........ezhdbDcafgCyix
tAuwyvxsz........gcaEiedfDChbBr
sDzyxuCrw........ihgfeEBcbtavAd
yztxrCwDu........cidgEfABshveba
AuyvsxEwt........zdbeBgfhCDrcai
vtrzuwyxs........fgAChBiEdbDaec
BxsuztrEy........AbeDCdhiacwfgv
xywrAsDvE........heBzgCbdiacutf
jpmBlqonChbADidca........kEegfz
CnAEkzpoDfeabgBdc........lmijqh
monkDjAlqEhzdciaC........efgBpb
DqkCoAnpmacfBhgbz........Eelidj
zmoqjDBCneadhbciE........plfAkg
kBCoqmjApbigcDafd........hnEzle
njqmCEzBlDfieabgA........cdkhop
EkDApBlmjcdCafzeb........gionhq
uCpjwomtxgvhrefyidEDkqbal...scn
qExlnpsyvdDewthCgmjcifakb...ruo
ovlDEyujkirsxwehfbpqamcCg...dnt
lrvptnkqBxgcfsuAyoazhdiemjwb...
rlBsmkqzoAxbgdvwepficahnjyut...
pwjnvltkrysuiBxzhaofbcAgemqd...
```

Type $9^2 6^1 4^1 3^1$

```
.........xtDvCyEAsBoklmrujpwznq
.........tBzDxusyEkqmpjnrCvoAwl
.........EvwAzDtsCqpBkoljnxuymr
.........zxtBACvEyDmqrpknolsujw
.........DAuEwvzBtnCyjrqsxklmop
.........vwstBzDCxAjonEplrmqkuy
.........sCxzutAvwjyEmDoklrnqpB
.........wuysvxBDAznrElCptjkoqm
.........BsCwDAxtulkzqyjmEnprvo
vEtByswuD.........gbfzehidCaxcA
swyDCtuAx.........hBceziEbavfgd
ysvtDAEBC.........efgcazxuhdwbi
wyBxzuAtv.........EaDbCgescidfh
tzuyAwCsB.........dEbihcvDegaxf
ECwzuvtxs.........idADBafhbegyc
uvECtyDwA.........bieadBgfschzx
CAsvxEBzt.........yDdgibcawfehu
xBCsEzvDu.........cAhygfwetbida
zoAknmpqrhEdyaecfb......DgiClBj
DplAkjqnyCeghcbfda......oiEmBrz
qlmErCzpoAycdbfegB......akDhjin
AqDrjokmzeaBfyiChc......dpgEnlb
lnzqBkrypbDACfEaid......hmojceg
nropqlmEjadheiByzg......CcfDbAk
rjkopDnlqgfEihwuxeCcadbm....vts
oDjmsrxCndiaugcbwvflphqE....tke
muqjwxlrkicvgEadbhoenCfD....pst
pkxulqojEchibtgweDmrCfnd....sav
kmnwoByvlfgbxsdhaipzjAcetqur...
jxrnvpsomybfcehiuzaglBkAqwdt...
BtplmnjkwuzeadsgcfrhioAybvqx...
```

Type $9^2 7^1 3^2$

```
.........wyECuBsvzpqnDrmAjokxlt
.........tzuvEyCAwqmrBpnlkDxjso
.........BEwxsCzDyrloqnjktpuvmA
.........sCyztvxuDjnlEAkowrqBpm
.........EAxyztDsCmrkloBqnjpuwv
.........xDtuBzwysAEmkCojlnvrqp
.........AuCtDwyxvzBjpmrnsElokq
.........DvsEyxuBtlkCnqAmowrpzj
.........CsADvutEBnzpmjlrxqowyk
wvzuCEDtx.........BAgfhebascdiy
AytxuvwBz.........DdehcbfECasgi
yADsvCtzB.........Ebfidhcugwaex
xDuEABzsy.........Cchdeagivtbfw
uBwCDtxys.........fhAazgEdcievb
sxyDtuvEw.........diBgazCebhAcf
BCxAszuwv.........aDEeidhgyftbc
tusvwxyAD.........bgczfEaheCidB
vtEwzsAxu.........ifaCBcDydbghe
mEpnolkCjeBdAcfazb.......qiDhrg
jzqBropnkgiaedcbCE.......DlmfAh
kqnlpmBjCdcebiDEhA.......rfgzoa
CoAzBqrDpfghceEiba.......mknljd
orBpEACklbfDiadegh.......cmjqnz
pjoqnDErmzdgBfbAci.......Chekal
DmCjqkolAcezhbifdg.......paEnBr
lkvoxwsqEytcgCadefhjDrbip...mun
EpkrmnloqiavwxhgfucebjDCd...yts
qlrkjympnhwfagsvtdoCibEDe...cxu
rnltypjmouhbswABiegaqckfzvxd...
zsjmlrnvtaxBfAehwckpdogqibuy...
nwmykjqurvbidhgcaxeozAlpBfts...
```

Type $9^1 7^2 6^1 3^1$

```
.........twuAryvmopzlkxnsDFEjqCB
.........BxArCszjlyknEoDFuwpqvmt
.........CvrswuxykmDjozplFqtEABn
.........DqxtuwyzECBFAkvnjrlpmos
.........zytvsFqxmDCApBurEjwonkl
.........usqExzrABlypmjFotDkvwnC
.........rBCDzqtFjxAonElmwuskpyv
.........EAyzDrCoxFlkjptqsnvwBum
.........wCsuFBEkpAmyznqvltoDxjr
wrqusvCxz.......hyBFicAEDagfetdb
zxFsDqywB.......gAbaEeCfdciutrvh
qwCrAxtus.......dihfaBycbvEDFegz
EvzDtwuAq.......BFdxhgaecbsrfCiy
uyACBFqtD.......fdEgcabrwivhszex
AuBvrywCt.......DbzExhFsgeacidqf
sFtBqDxyv.......EfgbzCdwureihacA
CDkyzEpjxAFaBbed.......miflgncho
FmDzyAnBCechagEi.......kfpobljxd
jzykxBDnobeiCdAf.......hEgmFclap
mCjxnzElyFDBeifA.......okdhabgpc
nBxlCmkDAyhgiEcF.......bjopdafze
knpAoljzFfdExBCD.......ahmcegybi
xonFECBklgzcfyDe.......dphbjmiAa
DsEtpnrFuqavwcdgiekomfh......blj
okvjutlqrdEDFhiwcnapebm......sfg
vElwFkmpjsuecathbgfndDi......orq
rjoqmpFEwvgbhtacnDedfil......usk
plmnksovEirdbqgueacjDFf......htw
tpsEjovrmciFqfbalhnegdD......kwu
BtwmluAokxbzdevspcihCygjaqfnr...
yquovrzsphfwgAxBaCjcbleitnkmd...
lArpwjsmnatfyvhbCzoiBxcgekdqu...
```

Type $9^1 8^1 6^2 3^1$

```
.........FysCrvuzmpjAExlwkotDqnB
.........yBDuECzAklxjoFnmwrqsvtp
.........DFCvAwxuqolByzEtsmkpjrn
.........vuFEzAtsjxymqlwrpDonBCk
.........CtxFwrvDEBzkpyjunsmolAq
.........BrAyutECFqDnzpvkojwlmsx
.........uzwABxsyoDpEnCqjmlFrkvt
.........AvrwCsytnEBDxjkplFuqzmo
.........wEBxvzFrCmqylDsotknjupA
txvrAFwCD........BzfbgchEdisayeu
vBtwsEyDx........zCAebaidFurfgch
BvusxtEry........eACihgFfcdDbazw
uswBtyCAz........ahgxcbDiEvdFefr
sAEuDzFxr........ydbfCegchtvwiBa
CwAEzxBtu........hiaFDfevrcbdsgy
rCszvDxyt........dFEcBAbhueagfwi
AExDrszFC........iycgeBabfwhtduv
qyklBApzoeCfDxFda......mgibEcnhj
pnlFyBqEkzxhcaiAb......oDjfgeCdm
mzBokCjqFgAyahDfd......pneElixbc
kDFCljopmcfibgyBh......daqneEAxz
xqyAjpmonEDdziBCF......feagckhlb
olzxnkABqdaEibfge......cFDhjmpyC
DpokmuvwEacesthbgfjndFq......ril
wrmqElukpsictFgDvbehafn......ojd
junvFotlsiwgrfahEDbmpdk......cqe
FoqmwnDvlrbufseicpkdhjE......tag
lFDjpvruwhdtgeEafcnoqim......bks
EkjpqrsnvthbeDucigaFlmd......wof
zmCyuqnjAbsaBdcrwxgeokitlvpfh...
ytpnCwlmjxgvhcdeBAfkzaorsbqiu...
njrtomksBfezdybwxlciCAhuqgapv...
```

Type $9^1 8^2 4^1 3^1$

```
.........FxvEDAtzmBpCjqokwursnly
.........ywuvrxzCkEAoDBnqmsFljtp
.........wDrsEFxAlCqpnzBmtjukyvo
.........AzFxvsyrjlkmqnpBDtoECwu
.........DuzByrswCklEpFjAoxmtqnv
.........tvCzAurDEqmBFoklynpjwsx
.........CAywztusDmElopqFjvkxrBn
.........vsDtuCAynzFqEmljpwxoBkr
.........ryxDtBwEpjnkmAzCvlsFuoq
sCuzrvEBw........gFcdAehDbiyafxt
zuFxvtwCy........BDghcaEbiefrsdA
uEwBxsytC........FgzcfhAdeavDirb
vFCuwDBsr........AehizEdgxycbtaf
trxvyEFws........hAdeBDCifbgczua
EwtrFusvz........bfeAgCicdDahxyB
FxyDtAruB........fiCzbgeasEhwvcd
wtBsDyvEx........icafCbghrFdueAz
CAEmjknzDigBfadcF........loepbqh
kqpFmnoDAzehadcEB........gfbilCj
jkDopzClFBfAdibec........aqEnmhg
DlACojmkEaFbcgeBf........nhqdpzi
ojqAzBlpndEfCcDhg........Fkimabe
AozlBCjmqfaiFbEDh........kpngdec
mDnqkpzolbcdhBiFa........EgjeAfC
BnjECqkAmcbegFzai........hdDfopl
rslkEoDnpeituhyvxqabFdjcf....gmw
xmsnuFqyoEhwrfvbtapijldDe....cgk
npktlwxqvurgysfibodDaecFE....hjm
qyojnrtFuxdEewagvcbfDhlmp....kis
pBmyAxurthCsiewfdznjgakboqclv...
lzvwqmpxjsBaACgduehobkifncrty...
yvrpslAjkgtcbxhCedoBnifazumwq...
```

Type $9^2 7^1 4^1 3^1$

```
.........tCAEFwxBDnqmzljrukyovsp
.........sAEwDFzyBmCrjqkpoluvxnt
.........yFBtEDwCuqjplormkvsnzxA
.........usyvCtEwFBzjkAqnmorDlpx
.........BtCFwuDxzrEkAjnoyqlpmvs
.........wyxDuAtEvjmzpkCBFnolsrq
.........xEFutzyvslkDBmpqrjnwACo
.........CvwxyEutAoBnFpDklsjrqzm
.........EwtzBsCuxAnoqDmjvpkFrly
DvzuCytFx.........cbiaeEgsfdhwAB
ytxsvzuBA.........FDdCEbhgeacfwi
sxAvwuzCD.........faBhciFeEgybtd
FCEDuBsxw.........bfeizdAtycahgv
vFtxsEwyu.........edAgCBbiDhfacz
usFzBvDwy.........EAbeicCxgfdtha
wBDAyCFvs.........giEdhazctxbuef
AuCwztBsv.........DhafdFcExigybe
xDuyEwvtF.........zgCcaAfdhesBib
oqmEDFjlrdgfizcaAC.......nbpkeBh
nzopFlqDkchgebBAai.......frmEdjC
CopjnrAzBfDcbedFga.......hmEqikl
ljBkpnEomFzegAhfDc.......qabiCdr
rkjlmqnAozcdChbBiE.......aFDepfg
BArmoplECabihgfcze.......DdFjnqk
zElFrAokjieDBdChbf.......pcqmgan
tmkrqsyupDxvfiaeFwdlgnbhE....joc
jwqnxDmrlbahdsvictkpFEfge....oyu
EywtkorpqeuasxgvdbhFfDnli....cmj
mnvotjpqEgibyfxshdaclrFeD....kuw
qpyBAkxntvdzcaebsgCohmrfljiwu...
krsqlmCjnhBuAvidfypecogzabwtx...
plnCjxkmzAfsacygehirqbBodwuvt...
```

Type $9^2 8^1 3^2$

```
.........CEzFBstxAljkoqDmnwpryuv
.........vFyDACstunmplEoqBzwxkrj
.........zwDAyxEFsolqjmBCktnvrpu
.........yDEvstwuCBrlnjAomkqFzxp
.........xvtCFuzAykDBqojrEpmswnl
.........DtuyCvBwxqpFmnkjoszElAr
.........FuxswDCytjqmAlEnrvkopBz
.........uyAztFxEDmkoBCrpjnsqvlw
.........BsvuxEyzwpAjkDnlCrFmqot
yAExvzBFt.........DhdgcCbafeuiws
zvDsuBCxw.........hgEFdiAcytbaef
uxvwEyzBA.........iCedfcDFbhtsag
BwCvDxAsz.........EfcibahgFuetyd
suFtzDECy.........AiheabBdgfwxvc
tEACywvuB.........fdbDFeihagzcsx
vsyBCutwD.........aEAchgFfxbidze
DytzxswEv.........ecfhBFaAuidbgC
AzwFstxyu.........dBDEghebcvafCi
jCqrFplkEAabfdgDic........eohnmB
qnBprCjAFdbeEcahfi........Dlgmko
kFponrqmCEcfdiBeDg........halAjb
FlnEkqmpoeBCchdbaf........jrDgiA
pBknAlDrjbdaegicCF........qEfohm
CqokpFrDlheigEcAba........mdjBfn
lmrjBEkqncgFafAide........oDpCbh
rplDoAFnmiCBbaefgd........Ejchqk
nkslmoyjxfzgtvwuchFbraipED...edq
EruylmntqwxhiDzFsvbogpkdfe...jca
xDzmqjolksicwehavEgFnbrfdp...uty
otjqwvszraAdxubghBceCfpmkliyn...
moxujkpvsthwBzfdebrnaCAqgilcy...
wjmAtnuopgfshbyvBzCairelcqdxk...
```

Type $9^1 8^1 7^1 6^1 3^1$

```
.........DCzAEBvrnqylFGokumtpwsjx
.........xGDCrstEkozAyBnuFvqwmjpl
.........AyxzuFBvjnoGmpCwtsElkqDr
.........EsGFzDrxBylmqCjopwnvukAt
.........BuwxCvADoEmzpyqslFGtjrkn
.........sErBvtuzDmpnCkGxojwqFlyA
.........txBuDAwyEFCkzjmnvprsGolq
.........vDsGFwxAyBkEnlpqjoumrtCz
.........urtswzCBpAqjGoDvxlFkEynm
AvutyGxFs........zCihBEegwdcfaDrb
ECBuAyFvr........GzDbfgcthisaxdew
urysxFEwt........CDgehAzbiavGdcBf
stCAzwyxE........FGcBbDarduiefvhg
rFEywCvAB........dhbDgcfiaGextzsu
tsDwurCyv........cehaAdFfGExgibzB
vywBtusDz........ecafEFAGrxhdbgiC
FwxzCEAuD........afGydhBestgrvibc
nqpGBAkzmdeFybica.......lEgfjhCoD
yGnEpkzBAcdebaCDi.......jgqoFlfmh
BpjDGzlkoabhgAdyC.......FfcmEenqi
zAFolBnjqCfEDyeaG.......cmbkighdp
DBlCkqmGpizghdyFb.......EenjocAfa
CEolmjDnGyAifgazF.......dkhpbqBce
mzAqjDpEFfBCdcGie.......hbklnoagy
pxsmqowtuFgvrhEGcibfdknl......eaj
ouvFEnqmxetbcGrsghajilfk......pwd
qoGnrlupjgFavfhewbkEcimd......xts
jntpsmorkGhdeixbfglFqcaE......wuv
xlrknpGswhcuaebEtfgdFjqi......mvo
GjmvFsrolwacEtfgdqinpebh......uxk
wmkrDxjlCzvytBuhsAdegoibpqfacn...
lkzjovtqybiAwxcfhmpBCDeganrdus...
kDqxvtBCnrwfisgduljAoazymcebhp...
```

Type $9^2 8^1 4^1 3^1$

```
.........xCAFzvsuwrDkpBGjomytElnq
.........ytGxuAEsDoqlFpnBmvwjzrkC
.........sBxuEtvzyljFkCqDArnmGwop
.........zvFwDCGxtjEBoqrAnypksmlu
.........vDtBwExGFkmCAjlqpsornzuy
.........wEzDyutBxqonlAjrFpsGmvCk
.........DGEtCFuvzBpqrnokljmxyswA
.........FxBzvsAECGlpqkDmjwrnutyo
.........EuyGxwFAsCkrDmpnBlvzqojt
EzusvCDwB.........AGcibaFegfythxd
swvuAxEyz.........DBhGdcCgiFbefta
utyFGAvCs.........faDBiEcdxzehbgw
CADvByztG.........aFEhedicugwfxbs
tDCzsFxvy.........gcdEhfeiGauwABb
zBsyxwGuA.........bhiCFgfEdtacevD
vFztysuGw.........iAegDBECbxfadhc
yGwxuBAsF.........hCfacebDtEvgidz
wytAFDsBE.........cdaegCGhzbivufx
mpnDjEBFqahfcbgdCi........eGlokAr
jqkGorpmnAdDCafheg........ElFbciB
ArmlEqkpCBideGbDgf........Fcojnah
ljABrmqDoGagEdifhb........nkpFCce
FoGCnjlkmdfabiDecA........qhEpBrg
GEFjlonADCgedBabfh........kqcrpmi
nklomGCqreAcaFBgbE........fdhiDpj
BnrEqpjokhbiAfGCFc........aedlgDm
oxqrkvyEpgcbstziwuFnmdGhaf....jel
xmjptuwrlcFsgedyaGniofEkhb....qzv
ksxmpnFjvtzuighwdaEeGcoblr....yqf
puEqwkrzxiyvfhectdmgjblFoG....asn
DCBnztoljbswvAcayeprgmfidkhuqx...
rlowCzmntuehysxBDvdfbjaApqcigk...
qvpkDltxufwChcyziBebAnrmgaojsd...
```

Type $9^1 8^1 7^2 3^1$

```
.........HsxuvFGyoECBzDktrjwnmplAq
.........zCGFsuHEmnyApBDxlkrvtojqw
.........vuCzwGsDqHAEFkBjomxtrlypn
.........wyBEzACrHpkDjoGlvumsFtqnx
.........CAHyDBxvGzEpqFjmstnukwrol
.........DHuGFyvxnjpoAEzwmqlksrCtB
.........rvAwtHEzDyBFCmlGpxkonqujs
.........ywFABszCElnHDpqvkotmjGxur
.........EBDxyvAtFCqmljnoGspwuHkrz
CExHsFDuy........BAzdgaefwGcrhbtvi
wxHCzvtEB........fbDeayFrudgcGishA
EszGHwyrt........CDcgBAfaeFidxhvbu
vHuBwxrtE........zhiGyCAsdabgfFeDc
ztBuCrFxD........AGdyhgEbiwHeavcsf
BAtxysEvr........beGCczHighuFwdfaD
syCwrtuDF........iBHzfGhgcevabxAEd
rwsDEBAFx........achfieyutvdGHgzCb
nDEmkAoCpGgbBhzed.......HFfqjicaly
kBozADlqHfehdCEai.......FnbGpcjgym
mlyjozBpGAFeHfCda.......cqihbgnDkE
AnDpqyCGkbzEcafFB.......hjHoldemig
lzAFpqGnmhDfaEcye.......dHgjiokbBC
DomyGjpACBhibdgcF.......naleHqfEzk
GCjqloHzAiEyDcdgb.......khnFfpmBea
HGvrmljksFcwebatugdfhoqi.......nxp
ukqntmsHjaxcherfwlFoiGbd.......pgv
pjltnuvwosGargxiHhqFkmcb.......dfe
xvklFGnsqubtfriwgpaejdHc.......omh
FmGvjkqolxfrsuebhcigaHnp.......wdt
qFwsxpklugivtHbrGdoanefm.......hcj
jpFkvHxmntrsgGhuceflqbda.......iwo
trnoBCzjwcadvADhsykmbEigexpfqlu...
yqrEunwBzedgCitDAjmblkhopfcsxva...
oupADEmyvdtzixwBfkgjcnlCqbrahes...
```

Type $8^2 4^1 3^2$

```
........tuyvsqzwimkljoxpnr
........rsvtzywumxjkplnoqi
........vywxrstqnulozpkjim
........yrswvzxtpjinkqmuol
........xquytvrskwzminoljp
........qwtzuxsrjkoimylvpn
........stxuyrqvwipzlkjnmo
........uzrsqtvxyomwnjpilk
wqvrtxzs........fgudybeach
stqurywv........adxfezgchb
vxytszuq........ecghdarwbf
tusxvqrw........zecybfhgda
rsxquwyt........dvabfczheg
qyzvwrsx........cfdethabgu
yrtzqsxu........gbwvhefdac
xzwsyuqr........vhbgadctfe
nvojxlpyhazfcdub....gmiekw
inlmpjvzafgcwbeh....oxykud
uoinzklpwdhafecy....xgbmvj
pljykmonexahbugz....cidfwv
zimkotnjbgedxhfclayp......
ojplinmkfbqedcygxzha......
lmkonpjizcdraghfbyex......
jknimotlgefbhwadupvc......
mwuplikocvbqgadehnfj......
kprwjvimdhcgefbaolnu......
```

Type $9^1 8^1 4^2 3^1$

```
.........tysvuwBzmjApqrokxln
.........xvwtsuArnzykplBojmq
.........stuzvrxAByljkmpqnwo
.........uwAsrBtyqoxzlnkjmvp
.........AxrBzyusolpvnqmtwkj
.........wszxBtrulkqomjnAvpy
.........vutrxAsBpwzmjoqlynk
.........zrButsyxvnmAokjplqw
.........rzvyAxwtjpnquBsmkol
uwABstryx........favbczdighe
tvrszwBuA........gifxhbeacyd
rtwuAvyzs........xgBidhfbaec
syxvurzwt........efdhAciBbag
wAtxvzsru........dBbyieachgf
zsuArytvw........ihecBagdfxb
Bzsytuvxr........hAgwafcedbi
yrvtxsuAB........bcwfgdzheia
lkyzqjopmBexdwvca....bgAnifh
AnBwmqxkoebdayhgc....zilfpjv
jBknlmpovhfyigdaw....eAbzqcx
njzqkxlBybAewacvi....fphgodm
mupojlknziBcgefbdAqha.......
olnrBpAqjcafebzhgkmid.......
qmlkpAjsnahibcgefzdoB.......
kompnBqtlfdhAiazbcejg.......
xqjloknmpygahfidvwbce.......
pxomynwjqgcbfdeihavkl.......
vpqjwomlkdigchbfeyxan.......
```

Type $9^2 4^2 3^1$

```
.........swAxyCBzvjkrnmptuloq
.........BtxzvACsumropnqjlwyk
.........zBtwCuxAskqplvonjymr
.........wvBtuzsCAlxyoqrkpmjn
.........tzwusvAxCrmBylkonqpj
.........xusAtByvwqoCzkjpmnrl
.........uyvsAtwBxnpqjrlCokzm
.........AsuywxvtzClkBonmrjqp
.........CAyvBszutxjnqpmrkolw
uCvyxtszA.........bwfedgBcahi
yzsvCuxBt.........agcbfdhAiwe
CtysAvuwB.........dhixgfabecz
xuwtsByCv.........zAhacefdgib
zBCwtxvus.........cyAhaiefdbg
syuBvwCtz.........eaxAbhigfdc
vstxuzBAC.........wfgdhacibey
BvAuwCtxy.........fbzcisgehad
txzCyswvu.........iBdfebAacgh
AkmpoyznqdeCbawgiB....jclhrxf
wplzjnrmkacghebidy....ABqCxfo
omrAnlpjxygficehwb....BCdqzka
mroqljkywedzagifhc....CAbBpnx
qlpoBrjknvhiCbacgfAdem.......
nqjkrAlsocChBfdbaipemg.......
kAnjmpoqlfidghcteaBCbr.......
rjBmkqAlpgfecihubdonaC.......
lnxrpmqojiabezgdfhycwk.......
pwklqonrmhbcdxfayegzji.......
joqnzkmprbxafdyecghilw.......
```

Type $9^2 5^1 4^1 3^1$

```
.........ADvtuxzByonpqjwskClmr
.........sutyDwAvCBpjzqnmroklx
.........DACvzywutrmxBolpsqjnk
.........xsuzvtCwBqjnkDmrplAyo
.........uCwxABysvnDlorpktmzjq
.........yvBwtCDzsjqrplonmuxkA
.........BtxDwvuCAmzyrkjoqpnsl
.........twsByzvDuAxknmClorpqj
.........vxyusDtAwkCzlpqjBnrom
xvCAsuwyD.........ahgeBifdtczb
vwyzuAstC.........xfBaehiDgbdc
yBuvwxADs.........fgcbzdaChiet
wAtBzvusy.........DbCgfcdhaexi
sCBwytzvu.........gidxcbDefhAa
CustvDyzx.........cBhiAfwabdge
zyDutsxAv.........CdafgBeciwbh
usAxCBvwt.........byDciegfdahz
ADwsxCtuz.........haedyvBbcfig
qomDBjrkpzcAdxbihg.....aCleyfn
kqxyjmColizhAbedfD.....rcnBgap
DrkolzBnAbyahCdcex.....gqijmpf
lpzmrynjqCaeiBAfxh.....kbgDocd
mxnCAlpBofdgeicabz.....Dhjkqry
njvlqpmrwhgcCfaBidekoDb.......
okjpDrqCnwBdshfgcilebma.......
BtqnokDlmcbigeshdaprfjC.......
tnorkqjpBdeDcgubafilmCh.......
pzljmwkqrehbfaixgcdoAyn.......
rmpknolxjaifbcgsyezAqhd.......
jlrqpnomkgfzadhetbyciAx.......
```

Type $9^1 7^1 4^1 3^2$

```
.........suvwqtzylmkpxronj
.........rtyzwvupnjxolksqm
.........qvwtszxkpoumnyrjl
.........ywtvzqsuoknxpjmlr
.........uzxqtyrjwlvnmpkos
.........xrzyvsqnuwptjolmk
.........vsqryxtljuwzompkn
.........tyusxwvomnjkzlqrp
.........zqsxurwmvplyknjto
uwvyqxtsr.......bzheidafcg
xqzswrytu.......vadfbhgiec
tuwrysxvz.......agebhqcdif
yxszuwrqv.......cbgiafdeht
ztuwrvqyx.......gcadfibhse
rzqxvyuws.......difacgetbh
vryqtzsuw.......hfigebxcad
pvknmoljybxihgca....dezufw
nklmxjpzowfadehc....gyivub
lypkzmoxnihebagd....jcfwvu
mjxpouznkecdgfiy....lahbwv
qlmjspkrthgbicfexyzo...nda
snotkljmqabcfrdhexyz...gpi
ksjoptnlmfdrchbizexy...agq
jptunkmolcigebafwdvhqrs...
wonljqvkpgafudebihcmrst...
omrvlnwpjdehaiugfkbcstq...
```

Type $9^1 8^1 4^1 3^2$

```
.........Avtzxyrwoqplumsjkn
.........uwvsrtyAqkxnmzopjl
.........xzuywrAtkvlopnjqsm
.........wxrAtszvjlkpyqnomu
.........zusrvwxylAojnpmktq
.........srAtyzwuxjvmqoknlp
.........vyzusAtxmnwqjkplro
.........ytxwAvuzpmnkorlsqj
.........tswxzuvrnpjyAlqmok
wrxystAzu........vhgceadfbi
rwzsAutvy........hiexdcbafg
tvswxruyz........gedhbAfica
zxuvtysAw........dgcfabiher
yzrtusvwA........ixagfdebhc
vAtuywxsr........zdbahecgif
Atyxwvzrs........efhdciguab
suvrzAyxt........bwfeigacdh
jpAlmqwkogcbhiaed....zfyvnx
xqnmjzkopcAfiabdg....lyhwve
kyqzonplmihafbdce....gjAxwv
qmlAvpojnbgyaeifc....khzdxw
mopjklntqrahgfcisyzAb...eud
nsoplkmqjafiduehbcyzA...rgt
lnjoqmrukdiecgfbhAayz...tps
ujmknxqplfbdechgawoivrst...
plkqrojnvhegbdxaifcmwstu...
okwnpjlmxedcvhgsfabqitur...
```

Type $9^2 4^1 3^2$

```
.........sAutxwByzlmjqknrpvo
.........twAByxzsvkqlrponujm
.........xzBAvsytuqkwmnrjolp
.........yuxwBtvAsmonpzklqrj
.........wvystzAuxBpmnjlorkq
.........AszuwvxBtnjpyoqklmr
.........utwvABszypnrxlmqjok
.........vBtzuywxAjlkorpmnqs
.........zxvysutwBorqAmjpknl
ytzuABwvs.........bxgcdahief
BsuxvwAyt.........cfzdaigehb
suwAyxtBz.........ehigbvdafc
uwstxABzv.........yghbifecda
vBxstzuAy.........ibfwehcdag
tyABzvsxw.........adehucbfgi
xAywBtzsu.........dabfhgivce
AxtzsuywB.........hcdifeagbv
wzBvuyxtA.........fecagbshid
qklypjomrBcbidhega....Azfxwn
mlorjpqnkiafcgAbhe....Bdzyxw
znrowlkpjhiceagdbf....qBAmyx
jrkmlqnoxbfiazehdg....cABwpy
kvqjmnlrpdhagifuecABoz...bst
nmpqrovklfbghedcaizABj...stu
povlkmjqngesbfiacdrzAB...tuh
oqjpnkrumcdefhbgiwxyalvst...
lpnkormjqagdxbcfvhwiyestu...
rjmnqsploeyhdcaifbgwxktuv...
```

Appendix 4. $n = 6$ and $g_6 = 3$

Type $9^2 7^1 4^1 3^2$

```
.........CEDHytvIFnplkBmAuorqjGwxzs
.........xyIsEFBGAkHpnrlDqwtvomzjuC
.........FDzEGvAwCHjIponrymutsxlkqB
.........sBvtHyzuIrqmoCpjFkDxGlAEnw
.........vGtIBCHxwzErAqFpnuokyjmslD
.........BIwDzuGytlkqFjACHnEompvrsx
.........wsyACExtHmznjIrBGpvDuqolFk
.........ExHuwsDBvorFznIqjGyptAkmCl
.........yCsFIwEzupmAGDklxqnrHvjtBo
wxtAzsCyu.........DdcHebIhvGgBaiFfE
tHsxwzyvC.........BAdDGEFfegbhcIaiu
HuzwFGBCs.........EDhaAeiItbcgdyvxf
uzvsCBGtw.........fhDeFgbEdxyIiHAac
vFyutHzIx.........bcgBhGaifsEACeDwd
IBCvEysGA.........dFHhcaztxwfiubgDe
CtwEuIAHv.........FeGbzDdagisxBhfcy
yDuzIvwxH.........GgBCEhceafdbtsiAF
sEDFvwtuG.........ifzIdBHchaeCyxbgA
DoGqmEljpdFBaiACcf.......kHIhnbrzeg
pkEGnoFBIeAgzaDbCh.......miHldfcqrj
ojrBHlEqFDfbCAache.......pIdGkzgnmi
EIqCprmDoAHeidcfFa.......gjlnzhGBkb
AnpmjDqzBGeibhgFad.......olkIcHfCEr
GlAHomjkzIiaBcfeEg.......bDqFrnChdp
BGnIrCkEjfhAeDzgHi.......lFcmqodpba
kqxrGtHFlhdEvgIsDbeijmacf....pwuyon
nwFlyuroDgbfxsHtiGhIeqpdE....vkacjm
lrmyDkuptHwFGbxIsEanocijg....feqdvh
FsojqnxrmbahyuidgDcGElHfk....wItepv
rvlpAxomqzcuhfGaeBICkgbindsjH...wyt
xAHtspInkazdwebifcgBCrloGvymj...uhq
jmInBqplrivGgxdyAsCabfkHhwceu...otz
qyjkxFDsntucdvehbzAlfimCorEpwagB...
mCBokjnAEugxcthwvyqbadfzeDrFilsp...
zpkDlAvwyctCfFBudxjoiEgqmsbhaern...
```

Type $9^2 6^1 4^2 3^1$

```
.........GwzDBstECojFqnAvHIklrympxu
.........sAvFyDzGuIECjHnmlkprwBqxto
.........BECGAvHDxkplymqstwuzjoIrnF
.........wBDuxGyFvnApkrCEqmtHzIosjl
.........DCGvswxAFqkjlBmHupEyIrnzot
.........IGwsDAvxErmHFpzqntloukBjCy
.........xutIHzsCwGlqnDorEvFmpjAkyB
.........AvEzFxwHBjqnoIkGDrsptuyClm
.........vxFBGHuIApCErjylksntqmwozD
AzwIEDyxC.........hfdaebusGicHgtBFv
vwsBtHIDE.........gyzAbexCFauGifdhc
uxFAItwvH.........yzBDCdaiEeghsGbcf
ByusxItAv.........CGaEzgfFDbwecHidh
yuCGswxIA.........DbhiFBtefdvgHzcEa
tIzHyEuwB.........dFAcgGhxefbsviDaC
wFItuzHyx.........AgGecEDdbCBfhvais
stAzFyvBD.........aiIhEfCcuHedGxgbw
FDGEBCszw.........eHgIycbahvxiAutfd
IrlyokDmjCgfizeFbH......cGdhqBnaApE
lHqmABoFpEdcghiGaI......ejCrknfbyDz
moEkGjnClFeiyafbhD......pgHIdczrqBA
rBDnHpFEkdcIaCyhzi......jfqoGAbelmg
CqyornGHFiadbcBEez......gIjDhmlkfAp
okHDlFBGmhzyCEbfie......IpcqAadgnrj
xvoCDGjpnftecwuisaEhmgdH....IbqlFkr
HmnFvupqIeibfdaCtslDcGor....jxwhEgk
EptujolkraHswiCIgfmcDbGF....nvxdhqe
GlrvCsEnogFHhbcDudiIfmqj....aktpewx
kspwzAmlGHIadetcygfobBhinrxj....vuq
qAjrwxztubyBHIgdfhcaepilkvoG....msn
pGkjmqAszyhgeuIBctbnrHfadolx....wvi
njmpkvqrscfAxtdawGHByzlIohig....ueb
DEBxqmCoyzsutghevbFrkdApiwncflaj...
jnvlprkuqtDxEfFAByzdiCahwbgmsoec...
zCxqnlrjtubhAvEgdcBeofkDFmawiyps...
```

Type $9^2 6^1 4^1 3^2$

```
.........uBsxvFCAzjlqkGDnmwptoHryE
.........GxuAFwzyBmCEqplrkHvotsjnD
.........vDCzsuyHFrAklqmEtpjnGBxow
.........zGHFDtwBykrAmEqluCoxpvnsj
.........FEGsHyAtvBjDrCkuonqmlpzwx
.........AuBtGxECwpzrFlomsqHjykDvn
.........sHAwuDGExColnFpqrvmBjtkzy
.........HwEutBvzGFDopkACxjsyrnmlq
.........tyFECvxDuzmnAjGowsrHqlBkp
GztysEAFB.........aecHdCwbDxgvfihu
DtyxAwusE.........bHeBiFGfhgvdzacC
uxvwDAtEC.........fFybHBgeacGsihdz
suCDEBxvt.........HyaghdFGfizbcwAe
CyABtvHGw.........gEzfDhxiubsFedac
wsuFvxyCD.........AGHcBztdefighEba
tGsHyDBzA.........EdihefvFxwbaucCg
HFwEzsCAv.........cfdDbyagiGhuxeBt
FAzvHyDws.........iaBCgedEGtcxbufh
kBnrlopmzCcyDiGhFE......HjbdaegfqA
jConBpkrlgadfyzHec......bqEhFiGADm
zEBGqropjyeficaFhA......DHmlkndCgb
BlDojGrqmfzhaeigbH......cnkCpAFyEd
ynECrmzjkehgGaHdcD......fpoFABqbil
mDqAoHFkGEibhfgBad......jCcnlzyper
qvjpkCnuHDFtegEaGihbfdmc....rwolxs
xwGkmFvlniCDgdhtuaeqbEcr....fHjspo
vqxlpjsHohdaCbfDwtGnFirE....ecmguk
pkHtuqGDFxfwbECisglcjena....dhromv
AjFuGnmxqdtecwbsifykhzoHplga...vrB
lmrjFuwnyBAvHxebfhopGazgscdk...qti
rHlzxkqopwviyBcudsngmGAbhaFe...tjf
opkqntlBuabxdzscgeDhCyfjivrEwmA...
nomsClEyrcgzvhAfxbdBpjaieDtuqkw...
ErpmwzjtxbscBAdevCqigoynkhlDufa...
```

Type $9^2 5^1 4^2 3^1$

```
.........ywAvztxCBnlpGEHjmDFkqruos
.........xFGDCswyzpoHlnvEkBtrAjmuq
.........wvHCDxtGFEBkornmjszyluqpA
.........EztBxAFHurGDyjoqsnkvwmlCp
.........CyDsBwEAvkmlrpquHFGtjzxno
.........uCBtwDHxEAnmqGjsolvpyFrkz
.........sxvwAGDzyCkrmoEplqunFHtBj
.........GDCxEFBvwyAojltnuprHkqszm
.........zEyHtvCBGxpjADuFqrolmknsw
wHxzvuACt.........BhbFidGEegsfayDc
sGzvHADux.........fdFiyCBbwaehtgcE
vuCDyEzxs.........GiBHdFfctAwghabe
EwAGzvsBu.........bfeDghCFadiHyctx
uEBCDsHAw.........ayzxFetGdcbvgfhi
BAvxutFsy.........iHCbcwDaEeGzfdgh
GxEywFutz.........hbcBegdDCHAsvifa
tzGHCyBwv.........dFgExsiefhcubAaD
CFswGxvDH.........zEAdaihtgbuecByf
lqnAFrxzEDbfhcdGgC.....pHBkjmaioey
kBHjqzCnmaGEyhecFA.....lbfoxdipDrg
xjDpmoGqCBeczfhyaH.....rlginFbdEAk
FlqmpnyHrhazcgbdDe.....koiGfjxACEB
zDyEjkqmGAceFbfadg.....BrCHpxolhin
jswktDoElFBaeHuvbhcqGgm....ifrnpdC
DntsEpljFHdguacbifmeqCB....whGokvr
HrulnBEFktfhGvigedjDacC....qopswmb
qpjoBCrkDfhuEFaetslcinH....mgdGbwv
pvmtxGwoqgHdAiyshcFznkfaerb....jlu
rylnomtvAciFbsHhfaegxpqGkwu....zjd
ykoFrHmpniAwaGzustqjfebcgdh....vxl
omFuklnGpvsifygzwxHrdaAbchj....eqt
mtprAwkljegbduBfEioCyzhDavcsqnx...
nCrBsqjyobuxidEAcDgahfkmwpvlzte...
AokqljprBdtsgeCiubDxEhzfvnmyacw...
```

Type $9^2 5^1 4^1 3^2$

```
.........CADBvtsuzGmpFqEwnryjlokx
.........xyuvGEBFwqDmlonkjptzACsr
.........wBEFsyvCxokqzDlpmuGrtAnj
.........DvCsBFAytkrGEjwupoxqnzml
.........AzxDtuwByFjnmkpEoGlsvqrC
.........GuvzxDFsArlokECjtnmpqBwy
.........yxBtwvEzClGDoFqmskpujrAn
.........BGtCuwyvEpnAxljDFskormqz
.........uDFEzxGtsjBryAmokCnwplvq
sBxAGtECv.........dghbcuaDwiFzyef
BtzDyvusw.........AEdcxFCGehgibfa
tsFvwDAzC.........xyEeGdgBabifuhc
zxEstwBDF.........CiaAgGbucveyfdh
CAszuEFBt.........hcbfdaGviwyxDge
vwBGCxsEA.........zFydetfhDucbaig
uFyEvGwtx.........eABDhsdafzbcgCi
ECtyBuGAD.........azxibgefvFhwscd
DuvwEyxFz.........fbiGCcteBgAhdas
qjnolAypridefCzxbG.....DFcgEakhBm
xqoCmrDGjEazyfdceF.....BlihAngkpb
ADklqBnoybChxaczGd.....frgEemFijp
GnAqxmlypgFihdBbaD.....ocCjrfEezk
oypmDjznBhbdcEgCfe.....iqrFakGxlA
wkrnjCmludsaGhfgciEoFBp....qtevbD
mplksqvrEctbeFCDhandfgi....jGowuB
rGjFknqvoefgbciawBDClhm....dEsptu
FmGBokpultEsaihegfbqCnr....cvdjDw
loqprskxnFwGuAafdbiegjzvhEm...cyt
krujpFtmGscywbAhEvgfzqaeild...nxo
yEmxFlrwqzgfAeGuihcpjanksdb...tov
pvwrnzojkaiAgybtDcmxeCfhBqlsdu...
nzCuApjqsfhciDedxgBakrybvwtolm...
jlDtzoCkmvewdgsiAuyhcpBrnbqfxa...
```

Type $9^2 4^2 3^2$

```
.........tAzByuvwFqEjmnkDCprlxso
.........uBsACywDzjklEvmFpxtornq
.........DxtCvFABErpkomqsuzlwyjn
.........wsBFzEDvtAonkClurmqxjpy
.........svytwxzCAErDnFpjluomqkB
.........yuvwDtxFBpzAlqoCEkjrnms
.........FCAyEsBuvxwrzpDkjqntolm
.........EtCsFAuzxoqmDBjnvypklrw
.........CzwxBvtAskjopDFlqnmEuyr
uwxsyDtzA.........eiCBEdfhvgFabc
DuAtwsyEF.........bgdxfeBichzCav
yDwvsuEAx.........zCBdicbgFeafth
zABwuvDCs.........hxiycfabEFgedt
xBEyFzvDt.........fhbwaCiegsucAd
vEsCBFwty.........cDebugAfaihdxz
sFuzDxCwE.........yBgfhtdAeavicb
wtyxzBFuv.........idagsAEDhcebfC
FxtuCEsyB.........gbceAivadwDzhf
kjqFlwomrzhEbiacxf....enpBDydACg
lCkDmynxwhdFfbgEea....rBcojzqpiA
CnlkxrBjDiyahAzFbg....oEqcfdpmwe
oypAEqjFzxfDadwCcb....lhemrkngBi
pmvEoCrqkBchuafdgiDnFA....sbjtel
nrCotAkvmeDbgfcaduFlph....iEsBqj
AvjqnomBpfgictCeshdFEr....lDbkua
BkmnvtlrqbFdEeDihCaAfj....oucsgp
qsFmpnuojdEeDhibfwlcyagrtk...vzx
jqnlrmxpugicdsefEDwyzFbaot...hvk
EzDjkpqlnaexvgdsiymfhctbrF...wou
tozrqkpslAwuicBhaenmxCjvgdbfy...
mprBjlAnocafzxbgydCewqkuhstvi...
rlopAjzkCvbgeuhytcBaqidsmnwxf...
```

Type $9^1 8^1 7^2 3^2$

```
.........AFwyzKIroDGmnCEpquHtxkvBJlsj
.........uDJHBAFsjnopCyqrkvxIwKtmlGzE
.........HCBIDryEqJAFpKoulsvmntwjzxGk
.........CwsEAyGxHpDzqBKIomuJjrkltnvF
.........rAEDKGzCnBmJjIFlHqtpvuokwsyx
.........xErwuJKtFqCGIAmvjonlHpzsBDky
.........wBIAtCszJEpylonKmFqkujDrvHxG
.........yzHJEBDvAkjlKGInuprFsxqwotmC
.........vxGtHEuFmAzCykDopKlrInsJjqBw
EuJwyzrHx........idFhbaAsvteKGIgcCBfD
vDHtrCyAF........GcJEgdaxsefuhiIzKbwB
uzAIHrDsG........BKaifghcwbFxeJdyEvCt
CvxHusGtE........DydBhfzFIriwceJKAagb
xCGBwKIEA........yhHdzFJbegasrvctfuDi
DtruCwByI........bzgcFEHeJxsfaGKhiAdv
HrysIDFzt........cCEKdJBwxibGfahvgeuA
ysFAGxEvB........dHKIezCgtawhJcuDbfir
zjDGlAmnCJHiFyeEB.......qKkIbpofgchad
kJEFnlzKjIdABGieb.......fcHpamqyCDgoh
AEmKojlDHzyfGebCc.......JFIkdqgBiaphn
KAqmBFnIlgcbaCfJD.......iGdoekhpEyzjH
GyBCzqKFoDbaeJgAf.......HnjmidlEIhkcp
BKCyJIqpmafDzdFge.......GhnjcoHibkEAl
IonEqykBDcGChFzfK.......jaJdHlmeApibg
jwtlsGJxKiIFugvHdhfcbkqe.......ronmpa
wnkpjvtlJGuKfsHcIgibemhd.......xaroFq
lFsnKupJvthxcrwagkeIfimG.......bqdjHo
FmuvxJHwkKicdIhtGplegobj.......ansrqf
tkpJFojqrsevKxIwifghaGHb.......ldmcnu
qGjoptxrshJdgiavwKbnkHef.......muIFlc
oIKqvkuGwFreihsbJlafHcng.......jpxdtm
rxoDknAjzdKgswthuImlqBpyabcJvif...CEe
JpIkEmoCnbvhxcuBaejqADildfwKgts...yrz
mqlrABvoyEtuCaxdhzIinJDpkgfcjKb...wes
sBwjtHCupfgzbvDiAEGyoackmrlhqFdnxe...
plvzmEwkuesyrbdxHaoBDAjctihGngFCfq...
nHzxDpsmqBatvfcryCFkjElihdGgobwAeu...
```

Type $9^1 8^1 7^1 6^1 3^2$

```
.........ACwrEJzuplmqnGIojvFtsykHBDx
.........yDCtzIrEljBpmJHquwnsFAvkxoG
.........tEsFwBDJHCokjympvGqIuznAlxr
.........sBvIDuwzCnHApFJrkjmoElyxqGt
.........FwzAtCurIypoqkGJsnlHmxDjvEB
.........wtrvxsJBnzFDClkmqoGEHIuypjA
.........GAJDIEvFoHzlyCBwxujnrptqkms
.........rJyHsDtwBGIEoznuFmpxkvqCjAl
.........CHDEuAByjolnGqptrsxkIJzwmvF
uHvxGCItD........FbfgzAiEedwJhrsayBc
CsIFxztvE........fdABaHchGJebgDruiwy
DyuvzxrBH........dgEJAaCIwtfGeibsFch
AFytJIwHx........zBaibDhscfuvGCdrEge
rtszFGxDA........chyCifEgJHIebwBvdau
ywAIBtECz........GiDbcgfHdxsuaehJrFv
JIrsuFHGw........haCzeBDxEgvitcfdAyb
xAtusBGIy........EfdagievhFrcJHwDbzC
BGHqAEloFJhcfCgyD.......eabipndjIzkm
EmBnHJDypaedGFbAc.......kilghjqIzfCo
njzopyBlCIFAiJHah.......GgkEmdfcbeqD
zDCJIjnFBHGhyfebA.......ioEkalgmpcdq
GBFmDnCEJcygeAziI.......dHqhjfbolapk
IJpHnAyjkhfBdeGEC.......FmaclqoigDbz
kEqlyDzJmgbIhHdcG.......afeoFpjABCni
vlojtrkwIbuicdfsgmqJHEpF......nxeGha
FnJGrpvqouxtagwHiembcIjd......klfhsE
lpmrkvusnedExbiGfgFqIJca......hHowtj
orEklwqmjvsGgihdxaJnFHIb......tecufp
toGpjqmxrfIuwcvFHbEehkdg......aJisln
jkxwEHJnGdvbsrFIaqchmfel......uptoig
pqnBmoAktxcHuvyCsJDjfhbzlIiarw...ged
qzkCwlpusBaxJychvDeijdmAbnIHgo...trf
HCjAqmsrvDgazBtxbyIkelhofpcJdi...nuw
sxDyouFpqzieCarftAkGdBEjclhbwvmgn...
muwEvsozlirFbGagekAcyDnqjtpdfxBCh...
wvlDCkjAuEzfBhxedipgGFoynbrtqcsam...
```

Type $9^1 8^1 7^1 4^1 3^2$

```
.........uBxFGEtDpqjlonkmvHrzwsCAy
.........EFDvxwsuzoCyGkAHqmptrjnBl
.........FHsAvBGronmCDEqjpktulyxwz
.........xCvDrsAzBFqnyoGElwuHkpmtj
.........twzsBHrykpDGCljnFxoAvquEm
.........zGwtuDxHCypmnAlsEFkqoBvjr
.........vxtzHFEAqBypmCnkwrDlGojus
.........GEywzvHsjlnFkporDtCmuABxq
.........ryuBEGFvHzAojDmCtlnxpwqsk
HxrAwtDCu........FaehzBgvfdsyiGEcb
xDwsGruFA........bCBegfEtacdvHhzyi
DHGtCBxyr........diFEebcwuvgshzAaf
yAurtHGws........aDEzBhegxfvFdbiCc
BsHxzwyrG........EAbcdgaDhCiftFevu
uECHyvrxz........cfaBAFbGdDwhsiget
GyvDrCEst........ehzfiHFubaxBAdcgw
zCywustvH........AcdgFaBbeGhifxDrE
CnAykElqDdzHaihgF.......cGpejbmfoB
mqzEFjnlkDhcGeCya.......pioHbBgdfA
npmFqGzEByDgiAeaC.......fkjlocHbhd
jGEBpFqDyHdAchiCg.......lmbfkzeona
ElBkADHzCheGydbci.......Foqjpnfamg
ABkzHymnobgFfCdhc.......qjEGaelpiD
qzDnolkAjBabgyfdE.......iHemGFchpC
wvtospFGqgfiEbxueDjHdacC....rmnlkh
sFnlDkomEiudCtawxGgfbHeh....cjvrqp
provEqCjxscahFtiwlbGHfmD....egukdn
tmxjlupoFCvrbsgfGhEiacdH....nqkwDe
ktqpmnBuwcAedayzhgHojbGfxriF...slv
FjpuvosknwbBHfretmGgAlqidcha...yzx
rwFGxzvHmaifugABdyklqhjpensc...tbo
vkjqBmAtlesCrczbfndhDpiyoguEwxa...
losmnAjpvfrExwuDBieckqyzhCgbdat...
oulCjxwBpAtheDcvbfmkiEzdasnqgyr...
```

Type $9^1 8^1 7^1 3^3$

```
.........wDxuCBGzpymEFqkjstrloAnv
.........rwEGBstvonljqmDFuxzkpCAy
.........yArzDFCEloqGjkpnvswutxBm
.........utABzxrGmlopyjnwDEvqFkCs
.........BzvyAtEwFCGokDlmxujsrnqp
.........vuBtsAFCGjEqznyklwpmxrDo
.........CxtDErAsjqzyGBFlnompuwvk
.........AvFCuEBxnpjloGmqtrswkzyD
.........EGwvxDurCmAkBlqtpFoznysj
ywxtFuCrs........DagzbABdEchiGvef
rAGvwEDxz........fFcehgiuBCytsdba
tuvxBAswy........bziDfCEedGFcgahr
GtrwAszux........adDBEbcvFhieyfgC
vEDAzruyt........dgBaCFfsebGhwcxi
xCwsyzAGr........BcdFDhafieEvbgtu
CrtFswGEv........yebcAfgDhidazBux
sDFuxvrCw........AEebaihBGdgyftcz
DGypCBEkqedzhiabA.......ofmnFcjlg
FyBjDkqmlGEagczhb.......pCfAnieod
zokqnmlFGfBCAgeyd.......EcDbjaiph
jnCGmFoABcgyiadfh.......bqpkElDze
BFzoEnmlkhbDfGiey.......CjgqdApac
AqnzkCypEFibedGaD.......cgjlohmfB
okACGDBzpdyiFhbgc.......amnefEqjl
mBElqokDFircxvfsgebhnpdC...aGjuwt
usmnjpvoCtahregwBEDFdicG...fxqlkb
kporlxjqugeGaFCDicBfhnEd...tbvsmw
pxqmujtsnzhfErwvFkGyicoegal...bdA
lzuErqFtjxsdwbycegkpfmaAGov...hin
EljytGxvmbFgswcduhfnAepzika...orq
qvpkoynjADfsctuiazhCmlebrwBxgd...
nmlDptwBosCubyvxfqiagdzjhrkcAe...
wjsBvlpnDacedfhztiAkCgyoxbqurm...
```

Type $9^1 8^1 6^1 4^2 3^1$

```
.........suAyzGrDEFCmHkvBnqwxtpjol
.........ystxwCHuGlojpAknEvrFzqDBm
.........CwDAFtBvjkEzlnGHrouypmqxs
.........FCHrvutyDnGxBlsmjkzqoApwE
.........AzyBCxusFGDpkoqEvwnmHtljr
.........wxBuszvtAEjqnHlGDCyrFokmp
.........GyxstFCzHpBDEmroluAkjwvqn
.........zFsvywGHxmpBjqDCurlAkntEo
.........DECwArsFlzmkxjBqopvuGHnyt
rwzysBxAu........cDHEGbFthidgavfCe
CtBvyDGEz........fiecbxHdFsgwruhaA
ArCwvFyHt........iehgcBEfdaxGbzsuD
vEDGByruC........aHAigctbweFshdxzf
sDurHtEvF........gCydiGehfbcawxzAB
xvtErHDws........zAcbfFCgeBGhuayid
HBruzvwFG........CxaAhycDsdbtfgEei
DzwsEuHtv........yBiCFhbrafedAGcgx
yGFHnqmxoeAdcBbDa......hkpEjzifglC
oqACDzFBkEhfgHdeb......jlGnaixcmpy
knHoAxCzlaDceiBEg......pjmGfbyFdhq
GoqBFmlyxbfedEAcC......apgDHjnhikz
pCyzGonqExbahgDAf......mcHFielkBdj
lxpACnBmjdHzEDebG......iFcghoqyafk
EHjDwGklqivuarsdBogFnef....mpcbCth
tlGkjCoDpgBwifcFhebnamE....qHdrusv
jmslqEprDBGFHdghwkobfaC....tnviecu
BFEpmkqoHutgbcvidnhlGCa....sfejrDw
mAntolzsyHgGFafxrpdkhqewuij....bvc
qsxFlrjpmvebtGHaAhyfodzuikc....wng
FpkmujvGwfrizehycdqxHognstl....Aba
ukljtwsnAhavGxyfimcdFzpgeqH....orb
wumqkpAjnrdhDbEzxBaglyifvCtocse...
zjonxstkrciECuagebfqyADdwBhpvml...
nyvxpAuCBtcrfhiwEqjzeDdoabmklgs...
```

Type $9^1 8^1 6^1 4^1 3^2$

```
.........DxFGByvujnmopqslktAEzwCr
.........rwDuGABFmozCnxEjqlpskyvt
.........wDtCrzAxFlopGEvsnmyjuBkq
.........uswvxCGtyzBljpqnrFkmEoAD
.........GrEFysxDApkqzjutCBmwonlv
.........szGyDEwrkBAjqClumovFxptn
.........CFBzArEvDmynxopwGkuqltjs
.........FtrxwBuApqGDykjCsnEovmzl
.........yuAEtxrCzDlGFmwqBsnkjvop
vtuGEFADx........CydcazbBwerghisf
GrxswzCyu........ciDFfdteEvgAbhBa
tszruvxBC........EfgdeAGcabiyFDhw
yAvzGtwuB........gCExhFraDcsfideb
xuDyCwtsG........dhFeBgfEvrbzcaiA
szwEDuFvt........fbaAgBcriGxhyedC
uDEtACBrw........xdhybiFfeaGvscgz
BFAvxysCr........acefiDgdtEzGwbuh
CpnxyAqEochgBaGiz......dkFDebflmj
zEkBljnxmbecAiDCG......oFghqpafyd
kyjlpGmqAEBdhbFDc......aioCfnezxg
DCqnBpGlEfbxgFezy......kmhidcAjao
lxoAFBknDeGzfEiyd......Cbjphaqgcm
qlpDnojmyAECazfhe......BgcdFiGkbx
FwlpqEuGvBiargtfhekCbon....jdmsDc
pkGmslvojdCbDfwgihFnBEa....cutqre
nqmjksowlhcedvbFBigpEDG....atrCfu
EGCFjDrkpvgfshdtwoabimc....lenuqB
jmtwonyFqiducsvabGExzAlehfg...rpk
roFqmkpjzxavtdusEleigcfhGbw...Any
AjykzrEpFtvsbuhcgnGqalemodf...xwi
wvBormlzkafyicgesqAjhCbnDputxd...
mnrCtxzAsgyiweabfBjckdhDvuqolp...
oBsuvqDtnzAheCcdabxfmkyipljwrg...
```

Type $9^1 8^1 6^1 3^3$
.........DsrvBCtzljykEFuomqpnAxw
.........wrxCEyBFqlDnkosutvjpmzA
.........uAEyxBCwjoqpFlvnDmtkrsz
.........yzFEtDwAmplCnqkrsjvoxBu
.........rBCDzsyvExjlAnowFpkqumt
.........xCytvwEunqADjBrmkloFzps
.........twsxuADryFpjBECvqkzmlon
.........zyAuFExtDCBoqkwlvrmsjnp
.........CEzAyFvspnkqxmtBluDjwro
wtxCADErv........dfFBcyhausezbgi
FsuytrvAD........BzcxagihEbfwdCe
rDswzvuBt........FeEfycdCAaxbgih
sAEFDyBtx........egdahCbirzucvwf
urzvECtFA........cBegDfabwdihsyx
ywCAruDsz........hExFbifdecagtvB
CutrvAsDy........zahbfxFgBEwdiec
tvBDxsFwr........gibyzAecahEufdC
jzqEloknCfaiFdegB......mApxcDyhb
nxDpmBzqFedbcChfE......Akjilyoag
xnkqymCoBdbcaAfzg......pDiFhEejl
ElnxCkozmAcBfbiaD......jFhygepqd
oFmzpjAlkEheicgda......DqCfnxBby
AyjBnEmpligdzhxbC......qeoDFfkca
voAkBnpEjFDwesuhfbmicCd...grtalq
mBFjqtlCugiDhevsdAcoEpa...wbrnfk
qmltwpjvEcuaBfbFekhgdoD...nsiCAr
DEoskxqunaFvdwrebiyfmlzcpg...htj
zqwlsFxmpbturgaiyoDnhdeEjf...ckv
kpyoFlnxwvftsDcrhabzeijgEd...qum
BCrnuzyjohvgwidcxfkmAeplsbtqa...
pkvmjwryqsefbazucxACighBtnodl...
ljpuoqwksBxhgrtAiCdazmbnfceyv...

Type $9^1 8^1 5^1 4^2 3^1$
.........wstuECGzjpFkDBmqAyornvxl
.........zEABsvxrnDkywGjplqtuFmCo
.........uzCAtDFxlmpjqkvnGsEwoyBr
.........ryDxvECtGwBoFAkljupmzqns
.........vDrwFtyuAEzpGlCBknxoqsmj
.........CxEtGwBDzjmlkpnvqosFrAyu
.........BvGEwyDsClxFAjtmorupkzqn
.........FGwCABrvDkyEztqjuxnsmlop
.........tFzvCrAByxDnmouksElGwpjq
tFGxyDEuw........BhbcfrAgidvesazC
uBysGAxzr........wFcDCibaEgfvehtd
BCEturAsz........fawgcFGedibyvxDh
yrwDtFuvs........aAdxEChbfzeiGBcg
ruxEstwFy........cCizaDBAvfGbdghe
xvuBCyrAD........FgEehfsGawdzcbit
DxtzrwsGC........dieABvEcbhFayugf
wAsrxuCyB........efhGiEFdDvzgatbc
noqywBGkmeibDzFcC.....hpEgajlfdAx
qwmAoCzjpEcagbdeF.....nlDBGkxhify
zGlFBqkDxhbdfeawy.....goinmcEjCpA
AzpljoqBGbfyFxcag.....eiCmkhnEDdw
CpnqkzBxoDgcbaGEA.....defFlyhijwm
jtCGDvplEauecisgdhqAmn....FrfbokB
ElBkpGmCFgtvdhAseqnabo....jicufrD
lDFjmpvEnABharuiGocCfd....bgqtesk
FmAunsDokfCiecgbhEdGBp....tqjlrav
vEjmqktrlyhFGfzdabogwxucsp....nei
psrvExonjcdgiyhzwmGlqebfFt....kua
GqknlmFwtxrszuifEpyjhgadoe....cvb
kjopFEymvGausgftixendbqrhc....wlz
mnDovjlpqiABrdbhckzfaysguCewtx...
okzCAlntuswxyBevfgbqijmDrhcadp...
syvwznjqAdefhDxubiBoClcatrpmkg...

Type $9^1 8^1 5^1 4^1 3^2$
.........EzusDFwvjqxCpArnlymtokB
.........vxtAuBDCFpyjkosrEmqnwlz
.........CtByvwrDlxnpzkmFuEosAqj
.........sCEwFDxAzmBlqnvotjrkpyu
.........yEstxCvFkzwDoqpAmunljBr
.........rBDvAzEwCFomjsupnltyqxk
.........uAxFwytEBkqzlrCsjvDpmon
.........wurCyEszmnjADvklpqxFBto
.........zrwutACBqlkoyDjmFnEvspx
tusyvCrxB........EhAewFfDagdzbci
uAvtDBEsz........bdFgxaeChwcirfy
svtrFzDwu........afeBEbdhAxgcyiC
BwxvrsACD........iyzhcEaueFbftgd
rEFwtAyBv........cCDaguidsbzhxef
AxyDBvFts........wgCdicEabefrzuh
CDEsztxyr........eBhFadbgcfiuvAw
xCruEDtAy........fwcbdBFigsvehza
jknFCqmpxAwhzaeBy.....iDEfoldcbg
ymAplkCoEbFcBigah.....fneDzwxdjq
DyzBjnqmwagAibfFx.....pocCdeEkhl
EFjmAonDkgbyezxcf.....CqBihawldp
FqkxomzEnDyachdei.....lgfBpjbCwA
pjmqslBrCtfiDEaAuobdkh....cFgnve
oBDnprvlmedFfchbagAEqC....kujist
mrlkujontBsvdgiheAEbfF....DpqaCc
lnBAqpsjFicdaCvurhogEe....tkDfmb
npojxEkzlFefgrtdsyDiwmhcbq...uav
qlpokwuFjxizEsbgcDafynthvd...erm
wzuEyFpqohDbrecitdjaxfmlkv...gns
zswlnyjupcvCbBrfdxemiAgtqkaho...
vtCzwxlkqfaghduybnipcBeAjorsm...
koqCmuwvAdhexfszgpclnbjBtriya...

Type $9^1 8^1 4^2 3^2$
.........ruzDAwtypEkBClmqsnvojx
.........tvAzsxCEoDplknBmrqyuwj
.........vtwyDAECBnoqsmpkxrujzl
.........zBuExCArkojDqtnslywvpm
.........wEDxtrsvjlAynquopCmkBz
.........yDsCEBuwzAqxrkjnmpltov
.........EzvuysDtnmBojplCwkqxrA
.........xryAwtvDEqmjuokzCsnplB
.........CyBvrzwsmpEAtjDlkxonuq
EzBuADtvr........iCwgesbcyahfxd
yAtCruvBD........caxzdifhEwgseb
rtzDysAxu........CehcadgbviEBfw
twurBzxsy........AifCgDEehvdcba
zBswEvrCA........gxcifbhaeDtydu
vDwECtuyx........dcgazBsrfhbiAe
wCrstxzAB........ydbvEeiDcuahgf
xuEtwACDs........vhdeBragibfzyc
CvyzmEDnkBwdeabcf....pgqjolxAih
Bpvyjnmqlhegwicdx....bzAEDfCako
lyqpoCwjzDgxfBdab....hceAnEkmvi
kmxlvwoznfbiacyhB....AECdgjDeqp
DlkAsonmEufbidgezaBCp....jcrqht
osljzBpuCearchEfADknm....qdibtg
AqjnDkltmgiEBCuzahfeb....doprcs
jnpmlqBEoAChrueidbzDf....atcgsk
nEokxjqpvisCtgfrelbawDhcu...dmy
moDquyskjbxtdehgcfvlEiCrp...wan
qjCopmEwtshabvDxgeyikcudf...lnr
ukmvqljrpdcehfayixwznoAtBbgs...
srAxnpkoqadcgbvBuwjyhlfzitme...
pxnBkrylwcAfszibhqgvdmaotuej...

Type $9^1 8^1 4^1 3^3$
.........CrxsBADvjoqltpmwnuykz
.........DurytxACmqnBjlzskpwvo
.........tvAwCByxkzopqDnrljums
.........AyBvDurswxCzpnktmqolj
.........uwzCxstyDmBvAopkjlnqr
.........yDstArvBzkxnCqoupwmjl
.........wsvDrCxzlnyjmAqotBkpu
.........stCryzuDBjvqokApwxlnm
.........rztAvwCunplxBysDqmjok
szrBCtuxD........idAwgbyhcavef
vuCyzAxsw........aBhgDetdicfrb
ysvxuDrBC........bgzhitcadeAfw
uxBrtyDCz........Aawbdchivfesg
DAyusBvrt........xecizadgfCbwh
tBAwvsyzr........gCaefhbcuDdix
zvtAwrBDu........dcbysCfxeghai
ADstyvzwx........ChgfuBiebrcda
lwxDkjqmocideavfb....yzgCBhpAn
CkmpDnwqBfayxgied....hjlvobzcA
pmwnxojAkzChdbeic....lfaBDvgyq
qpDCBzmkAaecgwydf....nijlhoxbv
BrqlmuoyseAahcfgipbkD...jCntzd
jolsAmptnbBDuzgcefidC...qakrhy
nqkjolCpyBgbcfdzheADa...mrisut
rluoqktnvgcfBeaswhDjmbdC...ixp
xCpqnwsvjdfuihDbtolekrmB...agc
mnjkrClupxdibshBavwfocgD...qte
otzvlqnjmixwaubhgcypAkrefsd...
wynmjpkolvhgzdtaAqficeurbxs...
kjozpxAlqhbeficwryvmdasungt...

Type $9^1 7^2 6^1 3^2$
.........ByvrHwGCzElxDntImoFsqpjuAk
.........tEqvrsAHozyFBxkjGIpDlumnCw
.........FvyxIEqzlHpmnBDkuwsGorCjtA
.........qtxzEFHyBmIjkAworpDvuGsCln
.........GwtDvICAynkoxlurpFHEzqBsjm
.........xHswGzujpBCnIDoFltvqmAkEry
.........yFABxqsnDpzGmCjHwEuItvrkol
.........DIruzvFmkoBpAyltEGnwCsHqxj
.........uGBEACwkIjDyFopmsntlxHqvzr
uxsvAGyBq.......gEDFfbiacIHdrwezthC
tEAxvrDyF.......cCIGBHzsueqhbfgdwai
wCrtEqIzB.......xAbHaGfvsdDeFicghyu
zsIrHvtED.......FdyxCieGabhcugwABfq
sDqCIuzAG.......aeFghEbrfHvwtBixydc
BqEsuHvwC.......IcGeDzadiFrfhytbAgx
IBwHsyxru.......fGaiAeEFDqbgchCtdvz
EIlnFxHjydDeCcbB.......gGfakmAhpzio
xpClBmAFzcgDIbdi.......henjGfHkyoEa
omnDzkGxHhedaCAy.......IEifjgcblpBF
AGoICEBHmzdhFDgx.......epcklnjyiabf
yzBkjAomnICHdFaD.......ibhlEpGxcfeg
GjFApIloxCbEhayf.......HgDdmikznecB
kApyonjCIfiFedxz.......ElgmaHbBGcDh
jwHEGpkqseuIigDtbnhclfm......aovrFd
DnGutjrvkHfgqwidohcmEaI......sleFpb
ntuFqwslvgrcHfGEimehbjd......IaoDkp
lHkmDtwpracusqfvGjgdioF......eIhbnE
FkvjrlqtoisbGhueEHfaIdc......pnwgmD
pvmowFuDjbaGcthreikEdgH......nfIlqs
muzqyonGpwACfBrhdaxbHlkcvjgIe...ist
qlyGnzmIArBwtiHapbCjkcgfhvsod...xue
HrjzkCpnlshagucIBxAfeyGqdtibo...mwv
CyDwlBFsEvxfAetbhgiozpjnqkcradmu...
rFtBxsCkwEzibyegDflAchpmnouqjvda...
voxpmDEutAqzysBclFdngChbwaeikrjf...

Type $9^1 7^2 4^1 3^2$
.........qFwzsutlxEGymAproBvnjCkD
.........EAqFvwDzmBkpjGrCuoxsyltn
.........GCEsBFqjlApmnzDkrvutxyow
.........xDBGztrmpojCynuwEsFAqklv
.........tsAxuDzkomyjGClFvpqwErnB
.........uqFErGAyjzonDmCBlwpvtxsk
.........vwDyqAFECpzxkojnGtmulsBr
.........ByvuGrCxznAoElwjFmkqptDs
.........AtyBFqsCExDzojvunklrGmwp
rwtyzvsuq.......cDGidxafhBCEFbAge
EqywFrtvA.......BebxhigGaDcsdzfCu
tywFrCxDv.......fBgEbzdhiquaGseAc
wCrAxuFtz.......icedDhEBGsbgyvaqf
ABuDvEyzr.......FgiheafsqCGdxcwbt
zrCtuxvqs.......GdygFBeaEfDbhwciA
vDxswqGBE.......gahFACytcdrzfebui
ynBjmozxGFiCfEca.......elphAbkDdg
FjDlBGACmiagbxEc.......nekdhofpzy
xpznyjokFDcbAeai.......EdhgGlmBfC
kFpBADnGCyziafxh.......cmbEoegdjl
oGEklnBAxcbfeCzg.......dDjiypFhma
GAopEyCljbBdcDix.......Ffgankhzem
pkAxCFloDefGdhBy.......mbinjEagcz
msvoDtqFpCEewifGandBlgk....rcujhb
BEkCGpuwofvhgcseDFlmabi....tjdnrq
DlFGnsrjtgecqadvbhkCEfB....wmiupo
umqEklwsBrhtDdCfebFnGpc....igovaj
noGqszmrlaxuhteEpyjfcAbkgwF...ivd
qujmtwEpksrziAgdnGcafFhbvel...oyx
ltnrpmkEyzGavwbuoAfegdFiscj...qxh
szlujkpmnwdrCyvBhfDbiexgotqcaA...
CvszqAjnwhuxtgybdkacBlDopmfeir...
jxmvoBDyudgsrbhwAiClkcpqtaefzn...

Type $9^1 7^2 3^3$

```
.........qyrAvDtFlBpmknjCEwxsozu
.........wvCDrxzjnAkpEotqFlymusB
.........FAwqtvrBCzyomDupkjlEsnx
.........tsxuwAFnpyBzDlvjroEkmCq
.........CwszFrBmkpEjAxqulDnvtoy
.........sFywEBqpDlAkCmrtonuzxvj
.........vuBFCzsEmxonljpADkwtyqr
.........BruCAtyljExDpFwksqvnzmo
.........AtvxDqEkyoFCjpnwmsrulBz
EwvuqAstB.......DeFzxdigfbrcahyC
uCAFEDqsr.......xabhizyBewvdcftg
qvswxzrBF.......dgDCebcEafutyAhi
yxDqzuvEA.......bBfaFhCdrtciwegs
DyqxrsCwv.......gAefdiEacuFzhBbt
FEzvtruCD.......yhdcAxBsiaefgqwb
ztCruvxAs.......fFhiyagcDBEqbwed
jlBEkyADpgfzabCx.......hnemFodic
onlpykFjzbefEgci.......ABhxmDCda
xzjnlBmFEciDeyfb.......odChkpgaA
CjkDomnylzaABfEe.......Fgibhdcxp
mkoBFCzljfxcyaeg.......bEndDipAh
koyABFDzCEdgceih.......lmjfaxbpn
nBEmAlopxaDdghyf.......CbczjFike
AFwtmnBokeqEsuhDCiadlcb...gpjvrf
BmtjsEpvoDCbiqwdhcnefFA...agrkul
lAuCDtEknihFfsgwebcmaBd...poqrjv
tDxsjpyuwrbevidcoEmghnzfFq...alk
vrnkwotqyhEadxFuzfilbgeDsp...jcm
rpFzvqkxmugtbcsaiojDEyheld...nfw
psrynxwmudcqhBbvazCjgfkioAtel...
suplCwjrqyzhtdaAcxknBofmvgibe...
wqmopjlntxBirzuCAdgbceakhvysf...
```

Type $9^1 7^1 6^1 4^1 3^2$

```
.........FuxswytnoEBjlqvCADrmkpz
.........DFvCstuEwymozAqkrlpnxjB
.........uyzEDxBojlnmAkrFCsvpwtq
.........rCwtAvExpnzDjFBlouksmqy
.........vxBqCurkEzAFDsptlwyojmn
.........qByAuDvFlpxwCrtsmkzEnoj
.........BqsxrwAjDCknyuEmpztFvlo
.........EDqyFBszCApknojvtmwxurl
.........twAFBqyDxkjlEpnovrmzsCu
sqEvyAFrD.......wzBefgtcadixbhuC
rCzxwDAuq.......ycgFdBEsehvftabi
uvBsEzyxw.......gFhCAbdDfeaqctir
tuqAvsrFz.......bfdwxaeCEcyhDiBg
FyDurtvsE.......dhefzxCbiBgawqAc
EtszqCxyu.......BgDbahifrFdevcwA
DsrCxvEtA.......cywhiFBduafbqzge
lmyEkopBjfdgezCi......DFhbxcaAnw
wpAnojmCxyzEDcbd......geBiFlhfak
xjwplBokCbfighFz......cmAnEDdeya
ykpoCFwnmzgcidAx......alDjbEeBhf
CxFmBykzpwebaifD......nAjEholgcd
jwmyFlBAkcaCdeEb......fhnDoigpzx
vlCrApuDBiEecahqmnFogd....jsfbkt
ADtjmqnlvasdrEchiBbgCk....eFuofp
BolkursqtAvFbgacfdmDEp....njiCeh
pBjDsnColehrutiFaAfEbc....qgkdvm
nEuFjmtwrdiDzxgehbacpfloqk...ysv
znvtDuqEFsrhwydfpkjaeomicg...lxb
mFnwzEDpoxtufveglacyhijkbq...rds
orxqtklvnhAaBfzwCeidymbgpscuj...
kzoBpwjmsgbfvqrCAixlcehadutny...
qAklnxzjyCcthbsaemoiBwvugfpdr...
```

Type $9^1 7^1 6^1 3^3$

```
.........xBEDswqljkmpArnzvyCotu
.........quBxyvtwnEklCmoDpjrzsA
.........rtwqzuAxCynBEDmlsopvkj
.........zysEqxuDomwkpCvnjrlABt
.........AxvtCyropzEnklsjuDmqwB
.........yCzuvBDpmxAwokEtnsjlrq
.........wEqzBrCnxpyAjukvDmotls
.........tDAsuExBknozmprClvqjyw
.........squCtABzEjlynvpkrwDxmo
rBtvzExDq.......ecAahfbCswudigy
uDqEyCsvt.......aAwiexBbchdgfzr
qrDstvyzA.......CfixdwEugcaeBbh
wystCruqv.......AgdcDzeiBEfhaxb
vxEqrACtz.......dwDBbasceygfuhi
tszwvBACy.......bhadxDcfEiqureg
stvrAxzBw.......fDcbCghduqeEyia
BknyjDElCdfxwice......ozAgpbham
AwlDpyoEmbhgcaCf......zBienxdjk
zmxjEokABicydDbg......nehaCwpfl
ynpBDmjklCzafwgE......Ahbdcieox
pzClBwDyEgAbhfic......ajmoxnkde
lABCopmwkedhigza......jDfxEybnc
CuAznlBrDEvcgefhkaopji...mbtsqd
DqukmztopBsirhdvEbCjce...flagAn
EpomlqvjragtBADshibefd...Ckcnuz
kCjxuslnoDbreEtwgdhfiyqap...mcv
xEmoknqsjurDvbedcylCghita...wpf
olknqtwmuhaCyxsijefDEbdgr...cvp
noypwjruxvefAdabizBhmcglqkts...
jvrAxunpsfweachymlgzoBtqdbik...
mjwuskpxncidbrqzyBegalfAothv...
```

Type $9^1 7^1 5^1 4^1 3^2$

```
.........qBtDswCopjmxnrkzyEvAul
.........CDyrtxwjAlnBqzsEkpumov
.........xACBvsywnkzpmErDqjlotu
.........DzxyAvukECpwslmjrqtBno
.........sxAvutqBowEzpDCmnyrlkj
.........zErCqBADkyljomtnvxsupw
.........AqDEBrsCyzwmtplkuonvjx
.........BrsxCAvpwoklEnutDmjqzy
.........uyztxqEmDBCvlkjowspnAr
EwyqDvxtB.......edAfaushiCrcbgz
qrDCEAuzt.......ahceyfgBsivwdxb
rAuzwxBDq.......fivyhdbECctaesg
AzrExyvus.......iBhgebqcfaCDwdt
usqvtryEz.......cxaBiDhgdebCfwA
tyEDszwru.......xvebCgcABfdqahi
wuxtzEqAC.......bfdvDraecghisyB
yvnxmpDCwEcghbzf.....kAiadlojBe
DoBlkCAmvdecwyiz.....jfnpEaxgbh
xljnvkopAwaEbhCd.....cBDgmiyzef
zxoBAwjnmbCvfEaD.....hedlpgkyic
nDwoyjExpvgBicba.....eCzAlfdhmk
oBtsClzqkfihAaugEbDcd....jemprn
BCApoqmknhdfceDrzlgjb....tuEias
mqkrpoCsjetuzfgBAaEin....hDbcld
sECjuBplDtbiagfhdzmAk....onercq
lpmyqstwogfdercxhCnDEijbu...kva
CklmjDnvErswuiebycfxgadoh...tqp
vjpkntsorywedDEilgbhcCuaq...xfm
ptzuBnljychbgwdevmxaoAiqrskf...
knvAlmrBxauqsdycgjiofztpebwh...
jmswrukylivaqzhtnepdABofbxcg...
```

Type $9^1 7^1 4^2 3^2$

```
.........vtuxszAojlnyrCkDwBqpm
.........DuszqxtkyoCjlBrpmvAnw
.........rDwqvAulkCxmBtynpsozj
.........xrtyABCznkwDpjqlsmvuo
.........qCztxusBvjDAnomwrplky
.........tBACrqvmpzokjsDxuwyln
.........BzyvCDwjlxkpmAsrqontu
.........CsBDtwrnmpvlqyAujkzox
.........yAxsBtzwCnmrDkljoupvq
BruyswDCv.......xbicgzqdtahefA
xyvuwrstq.......DBbaiAzfCecdhg
rvqBACxDs.......uzgdbyfaihtwec
wuDrCyzxA.......bfvBaectqdghsi
ABtsqurvD.......ewfgCadzhxicyb
CzAwrqyBt.......ighufdDbevxsca
tAyDzvuqx.......CdwhBsrcbgfiae
vjoxklpywAfChuea....zcbnmBDgid
DwjmpzCnBucbiyvd....ofghakexAl
lxpCnDAuyaivfzcg....dkeoBbjmwh
yDwjumBAkzbgcdih....eopCfnlaxv
nmBpljkozhycAaCigDef....dtqrbs
osznyAjrmehaBfgqpiDl....cCbtdk
kpnlBsmzriqfbhdcAaye....oDCjgt
plrtmBojCdaigDbehAcy....sfnkqz
mklotxnwufgqaehDvcdjsCiB...brp
qCsvxktlnbwDdirBaompcghe...ujf
jnCqDtwkosvdugaxceBihblp...fmr
uqkzoplsjgxewbfydhaAntmivcr...
zoxkvnqmpcehrwsfyuAbtiajgld...
stmAjovplwdrecybfxuzqhngkia...
```

Type $9^1 7^1 4^1 3^3$

```
.........rzvywAqumpxCtnBlosjk
.........sqACrBywuznjmkvptolx
.........CBuqzsAjkmplrxtvwnyo
.........ArwxBtslzkjoCmnuvqpy
.........xwszuqtBpyknoCjAmvrl
.........tvzwxuryCjosnpAkBlmq
.........yCBsqwvknultAoprjxzm
.........BuqtArCzovmpylsnkwxj
.........vtxAsyBnjowzlqmCupkr
AxqCBwrus.......vhdiybzafgect
qyztuvCrx.......fAcgeBihwdbas
trvywBuqC.......exAziafbgshdc
yqCsrzwAv.......aBgcxdefibthu
zCyrstqwB.......xfadchAueigvb
BArwCszxt.......iaevdqycbhfug
rzBxqCAtu.......gywhfediscabv
vlnzxupBkibagdhc....AfjomCywe
wkunymxzlcAbaCvh....gjBedpiof
lmoujAkypfiCdvbx....hzgwBacen
uBlAvoykjzfgeaxw....mpbCcndih
jnxkAympogschfzaCibB...lqertd
sptlkjBmnaxyrhcdbgCe...qoAzfi
pjsBtronqhyifcCeAdxb...kalmgz
mtwvnkjoAqgrBeafdclCbih...usp
ComjlnsvrdehutibpwfABgc...kqa
ksAqoplCmbdtvifgheBuacr...jnw
xvpmzqnlwecdiyguobhakstrjf...
nujopxvsywhecbdzmlifrkagtq...
owkpmltjzuafbgeicvnyqxsdhr...
```

Type $9^1 6^1 5^1 4^1 3^2$

```
.........vuqsDxwkCzoBpltjrmnyA
.........sqCvzDuwnojtrkymBlApx
.........qxvCpAjoyDnlmBrutkwsz
.........rtAquyvxkCDnojpBmwszl
.........yDptBrACzjmknqlsvuxow
.........wsrpvBmAluzDCtqxjoykn
.........pABxCvzujmlykonDsqrwt
.........zryAqwCmvxusBpDkltonj
.........DCtwypxBonvjzAkquslrm
xvsrpwtqu......bihBazDyCefcdAg
ustyDrzBA......avfwCqdegcpxhbi
vpxtAyBuz......gbwhfceidrDCqas
prwAuvszx......haDgdftCibcBeqy
rAuwvCDxs......BecyibfdahqgztpJ
tBCuzDvrw......ycibheqsAdgpaxf
zuyvmokDCghicwa.....AbnBlxfjed
kwnCBuAlyiedzxf.....mgDoahbvjc
mnvlykxADBwaufd.....Chzjgoibce
CyABlxumndvzDac.....ojgewbhifk
DxkoCmyvlAdbBce.....aihznwjfgu
lmDsjzqkpeyhrtbcdAaB....fCngio
AtpqsBCnjhbcfrzDygie....okdmla
yjqmksrCBcfDihgozalA....tnepdb
sloztApjkaBfygqenmdb....CiDchr
BDrnxjmtofpsgeCildkwhabc...uvq
jClpnqworugxdbhkDBecisaf...tmv
qoBDwljpmCagbitnfevxdcrs...kuh
wqzjopnyvtcuadsfhxAkglmbier...
ozjkqnlwtxiehsudgbcyrAfmpav...
nkmxrtosqbzweAiljufgpychvda...
```

Type $9^1 6^1 4^2 3^2$

```
.........BupAqyCkwtsomxlvjnrz
.........AwuztxmCkjrnlqBsopyv
.........tvspuqBAlzjmyowkCxnr
.........qztxpvunBykCAjsmwrlo
.........zyqsxBvtjwlronACmkup
.........rAxtCsowvnyjqzupBmkl
.........pqwCrzAjnuBkxyvotlms
.........uCyqArkBtozxjpmlvswn
.........vBzwyCnmolAsrkqtpuxj
tBuCvAqpx......yadghcisrfbwze
qtCrwBzAv......xefipabchduysg
xwzqsvrtA......huifgyBCdbecpa
sAvutyprw......zfCBxdhgeaibcq
rCypxtwBs......gvuacAehbqfzid
AxwtuCsvr......dyzeqBpaichgfb
CvxloutzywfBabd....igkAcenjhm
ynAvjxklmegCcwh....obzifBdatu
motwlnAxjhecfzb....digBCukvay
kumjyovnzdxbgBA....ahCltwcfei
lmszBpjyCbdircafghk....onAeqx
prqxzlyCBsbAhefacmd....njgiok
BplmCsxoniafdgebzyA....krqhjc
ozBnqrCjkycheailxbm....pAsdgf
jkrBnwmqtapgifuedAcClsb...ovh
vjnArklsuCheBipcogbfqam...tdw
usjkAmBwofravhgilcCnpde...qbt
zlpokjumqxtdyscwievbfnrgha...
nqkypzoulgsvbdwthaxmecfjir...
wyosmqnkpciruvtjbxhezfdagl...
```

Type $9^1 5^2 4^1 3^2$

```
.........uBxAzlmkwCrqsonjptyv
.........qrtxuCAjvkzyoBlswpmn
.........zpyCxkjnBvArqmutowsl
.........pxswCynAzuBljqmotkvr
.........tAoywnvmCjpkBlruqzxs
.........xwpzrjtlmBnoCyAvsquk
.........yuCrsBlvAwonpjqkmxzt
.........BoqvtwzulnyjksCmArpx
.........syABozwCtmjxrpkqvnlu
xqtBsCouv.....egdiAhpzcfwrbay
qBvpuorAC.....fhwyexzidtacsbg
tuyovwzsr.....adiexgBbCcAhfqp
CyoqwzBpA.....guthibcexsdavrf
porCxuyqw.....AfectsadgvbBhiz
AlnxjvuCkecBty.....fiazwgbmdh
vkxutlnwzabdgc.....CmAiBhjyfe
lACvyxwtjcezha.....mdnfgBiukb
uvwzAmtylfgibB.....aCxkhndecj
jzlnmyxBtAvuab.....kechifCgwd
BCmjpqAnybsrcedxzfh....oiklga
zskmBApjnrheqgbyxaf....dClcoi
nxsAzrlkBCqbfimacgy....jpedho
yrBsqnCxpozhiAckbdg....elfajm
rwutCsjvqiafdphBgkblAmn...oec
sjAwltkmohdcpfvCBuaebgr...inq
ompknBvrugCasqiefbdchlA...jtw
wtzrkjqlmvfgohxiancdsybpeu...
mpqyokszxdtwevuchjligfabrn...
knjlrpmoswivudtbyxzqfheacg...
```

Type $8^2 6^1 4^1 3^2$

```
........xtvDwsBrylojikFEAuqpmznC
........EAuyztsqmFnwCBlrkpiDojvx
........zsBvAFrtEkpxymulqCDojwin
........CDxsrwuyizlnpEBvotjkFmAq
........DEtAFyxwlnCBkjsipmuqvozr
........uCyqDzwEnxBmjApFtorvlisk
........tzrxEAqsCmwpoFDkjlyinvBu
........qFCzBuAvwiyEmorpsDljknxt
AqEtBDFz........cCxawdgsebvryufh
tuCxsByr........hAFczDqfvawEgbed
sDBCyFtu........xgAhczvqbdEaerwf
EsuzDvwy........FadbxfAhgretqCcB
DxvuAqCw........aBbgfetdrEFzcshy
yEwqtuvC........zceDFgfAdhaxsBrb
zrxAvyuE........bfDeawhCFBgdtcqs
BCsrEwqv........DdcygxbeufzFhAta
onyEFpBlhaDdcbzx......ijCemgwfkA
pwlixCDFAedByEhz......kmcgfnbajo
mFnBlixjdwAEheDb......coakpyfgCz
CAjyzomnBgEfadFh......eDlckwxpbi
wkmoizApadfegxbC......jBEFnhDylc
nlpDwxjmFyhCecEB......obiAdfzkag
kmFsCjrDbBqguvadeofiAn....tcEhpl
viDmqnlscrahbfegAEkFBC....ouptdj
lBAkrEntsubFCqvDojafhi....cmdegp
jpiFnAkBfcebqgtudDmCEa....hsrlov
qtowukExvfFrsDcaphjzlbmgni...dye
FjtnklzigqcuvryefbEoDpdahs...xmw
xorjpsikevzwdafFgyhlbcEtDn...qum
uvzpmroqyxstfCdckwgAehanBjbli...
rzqlotpAwhgaxBCfjeikdyncmvsbu...
iykvjmsorbwcthgABpzdnlCufqxea...
```

Type $8^2 6^1 3^3$

```
........sxuBCywqmnApEjivlDkozrt
........wBACrzquklnyopvEjisDmtx
........vsyDqEtwCApkzilmonxruBj
........ruztysBxECkwmDqApoljinv
........qCDztwuryxoipnjlkvEmsAB
........yErsvtxAzowjDBkiClmpqun
........zvqruBCDwyxmilspAjtEnko
........ErxuzCDsipyBjontvwqlAmk
xyDqCBvA........dcbgfzusrEetahw
sxrvAEwt........DaCdehbfzguqBcy
rDvyswzq........AhdCceEBfugatxb
zBxstrDC........aghEAycueqvwbdf
qvsxrAtw........hDEbydzCBfaugec
wtuDyvqE........fBcegbdhsaCxrzA
tzwAvuys........bEDxaCBqhrfdcge
AsBtwqCv........czgDxEraubhyefd
BwkzimElfycAdxhC......eogpDnjba
EnjloiAyDfawBebd......Czmkchxpg
yjpiDkozBweExAah......gnbCdcflm
jCApnDkmcbwxhdzB......feEyoglai
DkyBlzxnbcEeagdf......AjimwCpoh
pEinmCBoaDdbgfAz......hkcxjeywl
oAtmBnrDuagdEqcbeilhCf...spkvjz
nmqEujlkCzthDbgepdafBA...crvois
vuECptmBAdsafDegnjzolc...hbikqr
CqnrkliuxevychEtgfjabmoDd...wsp
miCjqysxthbcearEokflnwpgD...dvu
klowEspjegCfbuvcxmindaDrt...hyq
irzuxojpgqBvwcsylbmAhktdaenf...
uolkzpnrhtfgAvyajeBcwxmbqdis...
lpmojxuidAhqsrfvBwezkgacntyb...
```

Type $8^2 5^1 4^2 3^1$

```
........swFvutrynjoBpEmiAqlxDkzC
........uryFAqsziDpCmnljoxwEktvB
........AsEzBywqpkxilmDturFvonCj
........EztCDAurynlwkBoqpFijmvsx
........xvDutrCBFmwkjpnAiEylszoq
........wxAsqFDCjvmynkrzEluitoBp
........FuwqyBvEkCzjxApDmnotislr
........DFvrxzAwoiBlEsumtpjqnCyk
xzrstEwu........dfbDvhgBCaqeFcAy
vFDAEBzq........bwcxdCarhugsyeft
yqztsCvx........DhFdAabcrwEueBgf
rtxqCuyA........weavBzdFbsDgEfch
wEuBrAqv........xFDgzbCfchtadyes
sDtzxvAr........BydECcFafghbwuqe
CwADqrFs........abEeftzuByvcxdhg
qrsFBDuy........ExfAcgeCatdhvbwz
znjEAypDCdhBawFv.....eiogfbkclxm
DyCiowmjvBgcfEze.....FAhdbknpxal
jokynlCwBDcezhaf.....iEpFvxdbgmA
EAwvzFxlygabdCBc.....ofekjmDhpin
opFCvkjBzAfygbxh.....DcEeinmlwda
ijBolzrnqCuEFetDcakmh....dsfgApb
AkqmutnzbesgCdEFlBiho....Dfpajrc
klEnjospcqBDhgftAzCba....meFriud
timjksBFhfCAEDbdzlepg....croqanu
uvywDpEmradhescxfognFjtkl....qbi
nsvxFiDkaErdwcyuepjfblhgq....mto
FupkimtEgyxarfebvcnoDdqls....hjw
lmnpwqoidtbxcuhagEyFefjsD....rkv
mxouynlCecztsvqghdAairBbjkpwf...
pBlrmxiotbqfvagsCAhzwukdnecyj...
BCilpjktfhewbxdAmgvcyqsnzoaru...
```

Type $8^2 5^1 4^1 3^2$

```
........uDBsCywqkpnExrliAotmzvj
........yCEzDBuAjxvnolsqmtwrkip
........ErAxwqDyzkovmCtjipslBnu
........vBqCADrzxEiowmnpjsuklyt
........qyDvExsuBikmpzAnoCltrjw
........rutBvAywozCpjEiskxqDnml
........tzCrqsADilxykpjBEwmuvon
........CtxEsvzBynDwlqropkijAum
ExBywsvu........fhbAeDqtadCcgzr
vsCtzqyA........DacBbfEruhdewxg
xvtquAzC........eyEaDchfgrbwdBs
DqyrsuCv........bBAhgtdafcxEewz
rBuwAtsx........CedzygaEhfDvbqc
srvEDztB........wdfCAbuheacyqgx
qEwsryxz........acgfhABdCDvbute
wtqzyxEr........vABgCdbcDufsaeh
ByjkxDmiwacbzfhg.....noelvECpdA
CzljBikDfgvdcwaE.....epmnbhxoAy
njmDECiwdvaAhebx.....kfgzypoclB
lpExnvDoAwbgeacC.....hzkBmyijfd
oAnBkEwyxdzcahve.....iCDbljgmpf
iusopkrlBAgedzCtmjhDE....qnafcb
zkoCmnjEgqfutdBsADlbc....eaphri
AlkmCrBsecuDgEdbpfjiz....noqtha
uCzltmApDfsaBgqchoejn....Erdibk
ynxpvjombEehutfrdgwliaDCc...skq
moDiqplncxrwfbEhgCaedjkut...ysv
jDiulwnkhedqyCtfEmpcvsgbr...xao
pmrAilqjashtbuevnwyxBoczdgkf...
twpnjouqzbyfrcxdlvmkaBeAsigh...
kiAvoBptshwyxrgacbzdfumlqjen...
```

Type $8^2 4^2 3^2$

```
........ADvCwzxynpumjBotksqlir
........wxtyvuqDmjiopnsBrlCzkA
........yvwtBDzqonpxlkCiumsArj
........stxvqBDCjmypizrlnkouAw
........CrzButAspkjwylnmqxDiov
........qCurxvswyoliAmDjpBkntz
........vyCzDxwAilkBmtqosrujnp
........rAswtqvBuxDjnCiplomkzy
wsvyqxrz........DCBAchbgdtfaeu
DqruzAwy........BgdCscfetbvhax
szBqCrDv........Acgyaftdxebwuh
qrzDwBCx........beAutyhcgfasvd
zuqwtsyA........ahxgrbBDCvedcf
uwDsyqtB........hdCfzraAbgcvxe
rAtBsuvw........CzfhqaybeDxcdg
vBCxAtuD........fwbedsgzachryq
xDipvzkldaBhgbyu....CAcfowjemn
mnAvlwzCaByxdhcf....gDekjuibpo
AkwCimojxufcaghe....DdzyvnBpbl
oxyljpAkczbueCBv....fidnDawghm
tpnmryiofsAaCcehlDzk....Bdgqjb
kosABDpnbfeqzdCrcima....hjlygt
BlmnpojiDhqAyrgadbez....cCtfsk
CjloknsrBqcgbAtzeyhD....ipdmfa
pvoiujmthbdDferckawnBglC...xqs
jtpkDCBuewrfhsdgxvncbqma...oli
iCkrmlnpueDdsfbxgBavhojq...twc
nmxjoilqgcabryftzuvdkeAswhp...
yijznvxstdgeAaubwfclopkhmqr...
lyutxkqmzghscwadvAobejprfin...
```

Type $8^2 4^1 3^3$

```
........uwvCsytxAklznioqjBrmp
........yrAzCtBuwmnlisjvkqopx
........qBxyACrsjzkvpmionlutw
........zAwxrqCyojpkslBtvminu
........xzCvuwAtiyjBqpksmnlro
........svBAtzuwknmoxjypCiqlr
........CxyrwAzBlivpmtnkosjuq
........rszwBvyqulontkCjApmxi
urBvCqyz........ghecAadbwfxst
vzruqswC........BbxahyAgdtcfe
AuvrwBts........aegfdxhcqCzyb
zAsqvxCw........cduyeBfrthabg
wvqxrtzB........buACaeghfcyds
CwuyzAqx........fBdbrgsehvtca
rsCAtzuy........xvcdbqawBghef
Byzwxvsq........eAagfbtCurdhc
koxBpjiuAafbgdhc....yCmlewnvz
mjozyiAnvCcexugd....lfpBbawkh
pBmCuykjefgdbawA....chxniovzl
ynAilkvmBgbhfxeC....zocdaupwj
npyksCBAcdrtzeqbhxfi...aljgom
xktjolpideacqbfzyCBh...mrAsgn
iCjoBplktqdshcxrnfzm...Agebay
tikpmnrlhusBcfbgCawAodq...ejv
slptAwjofheqaBcvmgbuCnr...kid
omiljunvgthadrsepcCwBAb...fqk
qtlnkomrbyugvsafzwhxjceipd...
lqnsimxpwbtfehdavoyjgrzuck...
jxwmnrotacquygvhdpiekzlfsb...
```

Type $8^1 7^1 6^2 3^2$

```
........FprAqCByjxGEosnDimkvwlzut
........pvszwGACiFkjytErnDlquxBom
........wExtpuzjmBCivqDnoGFslrAyk
........rxtBDpwiFAvzkEomljsnGyCqu
........ACFwEsrxBmovjDGitpqlzkuny
........CzwurAxvEDylnjqsmiGoFtkpB
........styxvDFmCzEBAolprkjuqGiwn
........BwpvFtDozkmGEiClunrysqxjA
DpvtGzry.......AbdFcauefCqBEhwgsx
qyxEFpvG.......fcwAhdbauBCDtgsrez
pCuBvqFt.......EehbyDfsgGrcAxawzd
BsyDqFuA.......GvazwxgphbdEreftcC
yqAptvzB.......gxbdfhFuCaseGcEDrw
uFtyDsEx.......zavBgGhrdpeCwfbcAq
sEwCxtDF.......BAchdfrbGegpzyuqav
ivjFAlowyafEhgG......CceDBmxnzdkb
wnmABjGlDdbhCzE......egakFfivoyxc
vGkwmAxCzcaDeyh......lBEdbiFjnofg
kDioExwjgBzyfab......GmcFldeAvnCh
CjoGlwBDxyhezbc......dkFfanmEAvgi
GzlvwonkhADfyxC......mFbEcgdijeBa
nrBqiukoGDEbshaedCcml......gpFjtf
oBCuknmrtegGcqplDEfaF......hbisdj
jmEkriCqfFupGedcloDnB......atgbhs
moFluBsEabqctrgkGjnCe......pdhfiD
EknipCtubsGrBcqhfegFm......jodaDl
FtpsjkqmEhCdaBunoGiDg......frclbe
AlGjyrisuqeFbEfagnwxzkdtcho...mvp
lwrznGyvqucsAdtFkgxeiafjhEb...pmo
xusmzEjpvGAgdFybnfekwcioqta...hlr
zxDnomAierdauvswyljbCBhqgftckp...
riqxsDpzcfvCgwedhylAbntBjoukam...
tAzrCylndgBqxfvDwiaocpjksuhbme...
```

Type $8^1 7^1 6^1 4^1 3^2$
........EzBrxqComDnwljskAituyvp
........AsECwvyDokmzntpuixrqjlB
........ypzqvtxnkjDmoEABrwCluis
........qtDpyEwBvCkAminzljursox
........vDuyABpEiwxkzrqCosnmltj
........trCBDwEjxyznvqklumpsiAo
........xvswEuDmyAoCilrnBpjktqz
........wxqtusrzBvlyjDCokEinApm
rxytpzsE.......ACcBghfubqdDweav
Atzvupxw.......gcedEysfDCaqhrBb
trDAvsuC.......bEBhxwzgfpyaeqdc
ysvwqBpA.......xgECDadztebhcfru
uqsDytzx.......fAdvBcbhEargCpew
ECwBsurq.......hDxAbfpedztyvacg
pvEzrqBu.......CdgbeAhtcDfsxwya
ilkoDnAvbcwxhaf......eBjgCdEzmy
jBmlxioncEvaCzg......kDhdewybfA
kolinyDBdgefzbA......CEacvxjmwh
BDAxwoCmhybEadz......gjinlvfcke
xwiykjEzDhaAbCB......mcefnlovgd
vkBECDlozwcdgye......amAjhfbxni
mjtnACklpudsBfqeboaiE....Dcghzr
sipmzEtrBbgucAadhlefC....koDnjq
zArsEmnifCpcqDuaehjdg....oktBbl
qzjCmkitrBADephclbfod....uEagsn
DEurlwjsCqygfeckamivbndph...oxt
CnqpilmDservthdywacjxobgE...kuf
wmCqtxyjgafepcbvniElDuors...dhk
nyoujrqpeAthdxvlfzwckBasmgbi...
opxkBvwyufhzsrtijngaeAlqbcmd...
lunjoAvkadxbrgswzfyhBcimtqep...

Type $8^1 7^1 6^1 3^3$
........uprsDxCBjnvAkoliwmtqyz
........ByzxrApikCnwmlquosDjtv
........CqAyzpsonmjvDrktulBwix
........DAwpyuvjBkxlitnqrCsmzo
........xvyDCtulzimkBApnqwjors
........rupBtsAzxlwiCDjokqvnmy
........pCxwqrzDmvkoABtysnliju
........vrDqpCtnljzBxmskiowyuA
AqswryCz.......vDBagbcuhtxdfep
Dtypxrqu.......eAagzhCdsfBbvwc
rvqsAuDC.......fweBcyghzbdpxat
sCwzvBrp.......hcxyfabeDdtugAq
tpxADwsB.......yhzdecuCfvgqrba
qBtuszvD.......AyweCgarbpfhcxd
pzuqwtxr.......CghDydeABacfsvb
zloCmAjxwbBfead......kycDvghni
woAykClvagfdbDh......nmeBixzcj
oxBDijkAhwdbgvc......yzCnamefl
vjzoyxmkfDceAhB......ialCbndgw
nwvmjiBobcagxzy......fDdekCAlh
liDvCknmydeABwg......zoaxjcbhf
inCBumtysehrdqfcobAaz...jDklpg
yAlkpqujgzbcsBraCdfDn...heitom
mukrolyqzBstfbDgiAChj...cpeadn
CmpnloitcfuhweaxdDbjvsBg...kqr
BDixnspwtaCvhgqdbcomljfr...uke
urnlBDoidtgCvcwmafhxeqbj...psk
xymjtpwnqsvzufbkegcdohiAlra...
jkriqvzsAxtuadebfylnwpcmgho...
ksjtznAlehqacyxwvoibfdgpmur...

Type $8^1 7^1 5^1 4^1 3^2$
........BuAyspDijmvwkCnrtxlqzo
........ArsuyCwlDoixntqzpBjvkm
........sDwAxuBjkvomznyqilCtpr
........xpDvqtsunizklBjoCmrAyw
........qAypwsuzmDBvrjCtlikonx
........ywrsBzCovxkljitnuDmpAq
........txprCAznijwDykomBsquvl
........CBvtzrqwAlDipysjounxmk
vCBrADpu.......ayzdhqsbecwtfxg
uvpDyqwr.......exbcBahfAsCgzdt
qDvzrwys.......bCBxAfacpgduhte
twqAxsuB.......fgaCdhDeyvcpbrz
pqxstvzD.......gayubCrhBdfwecA
xrtuvBqz.......dwhyCAepfDgbcsa
rtzwpyxv.......CBeAucgdhqaDsfb
zAkxoumladhCfgb.....ecBDwjvyin
yluizAjwhcCDvax.....Bdmkbeongf
jkwnimBCDvzcady.....goAbfhxleu
wxDouzAjfgcBbev.....dmiCnkhaly
lziCjkoyvfgbexc.....mADdanBwuh
DiCBltrkezqadbpAfnmy....hocgjs
nsymBoCtcedhpqfDlAgz....rbikaj
inlkCpDAzstehyrBofag....jqdmbc
BosynrimbCadcDAhzglf....ktejqp
sBntwxkqrhbfgcamuCejDpli...dov
CmolsjnpuqxgDBhvedbatfkc...rwi
oumvDCtxpaBqrfekcwjnsbgl...ihd
mpAjknligbuztwdyhcfeoqraxvs...
Ajrqmisnwyfxuvtcdkhoblzgepa...
kyjpqlvodtewAhgxbuncizasmrf...

Type $8^1 7^1 4^1 3^3$
........sqxtuyzkABwnmlovprji
........uyArqxsiwvjoBnzmlktp
........rxtqvsAonlmizBpjkwyu
........BsvpxryzkilAwqmotnuj
........qrwAzBptujoklsvnixmy
........xpBvtwqnmAzykiuljsor
........ywuBrAxjozksipqtmlvn
........tArxyuBmjnvzskipoqwl
prAszuBx.......fbehwactdvygq
uqzwyvtp.......BxadfcAehrgbs
Atuqpxvy.......hBfbrdegzsacw
sytuBqrz.......wvxcefhdAabpg
xzqAwpuB.......vetyhrasgbdfc
zAxyvwqu.......dtcaBegbsfprh
rvBptsAw.......gauxbqyhczedf
kjytiBwofdagezc....lxmAbuhnv
ywvzxkinAadhBeu....mjbcfgolt
nlmjuyotwBzcabe....gAxfidvhk
tuwvlizmegyfbch....dnoBkAjxa
woixmAkjphfzsdraygn...lqBceb
lBkonjyibzqdhpaAgwf...recmsx
mxnlsojrcbgwpadyzhA...kBqfie
qkjBrnlvzchsAgfuibepod...tam
BslnjrmAhtpafvbecouqgz...ikd
imprAznkvesbgftcldBahj...uqo
vnomqtsldubywhgxfkicprjae...
oprikmxsgfcedqvlhytjbwaun...
jiskolpqaveuctwbdmgxyfnrh...

Type $7^2 5^1 4^1 3^2$
.......rusxCzpkiwnAByomvqhjtl
.......upxryvtwAlCzsnhBmkqoij
.......vswptoAjmhunikxrCBlzyq
.......yqozrxuthmjlAipCBskwvn
.......ABzsqCxulvkhopyjwtirnm
.......qzBCAtwhkjlvyxropmnsui
.......zvpqxAyintwmrCBsulohjk
ApuvwCs.......ztxygdBecoarfqb
xsqBrov.......efytuCzdpaAcgbw
zuwrspB.......gCfeyqoAbdvaxct
wvBCpyt.......fxzceadqgArubos
trvzxAo.......yuBawfsCqbcgdep
qzpsvtx.......dyABCbgaecofuwr
rBxoquw.......AdbvtcfzaeCpysg
ljCxyBmgbcwzuf.....nAkihetadv
hCtulxjcwyeBad.....mbinkgAvzf
viAhmnCwguacdb.....kjlztfBexy
mAytCizadbfgwB.....ecjhluvnkx
Bmznuwhtyavbec.....xlfAijCkgd
oyhqjkrbACBpfsxeiza....gndmlc
yqskhzlBCrAdpgcanxf....jbeimo
nklyAsixrgoaqzbBCmd....fpjche
CxrpnhkfeAyogqBcdbj....simlaz
jtkAiqnsoecwrClvugBhmbd...pfa
iwnjBrACadbesovgchklqmf...tpu
knjwomudftgsBvCbeAipacl...qrh
phmlzvqocftuyanwgixjeskrdb...
sloikjyptvufbemzadcgrnxqhw...
uoimtlpexqdvcrajkfbzhgynws...

Type $7^2 4^1 3^3$
.......vApwytuhnixmzqoskljr
.......pquvAwrxmlzoyniktjsh
.......uvsrxpoAwyhzjikmlqtn
.......Atqzsrpijkwvlxmhynou
.......qyvxrowsuhtnklAzjpim
.......ruyqoxvtisjlAphnzmkw
.......zwxytuAlsnkrmhpjivqo
tyouzws.......aevdAgrbpcfxq
upysAqo.......zxefdvbtrgawc
oxsyqvp.......bdzAwfcaurteg
stzouxr.......vAwgypdeaqcfb
ywupxst.......eacbgqzdfAorv
qovtrAw.......cfgubeyzdsxpa
xzpAsou.......wtfcqbvrydgae
klAmjtycawsfzb....xhenguivd
ivtxmnkdzAuagy....fwjschebl
vslwniAgbaeuyz....cxkftmdhj
jkwviylfecdzbx....hamgAnsut
nijrhzvaxoAbdqkgme...ylpwcf
rqxzpjhwodavAgnlby...cefkmi
wAihymzodecpvfgkal...jqbrnx
zhmjlpibcrfeqsdytnaoA...ugk
muqnoljertbgfcyzAskia...hdp
Arhitkqypzgdeafcumjno...bls
hmnqkrxsgfpcatubjvedwlio...
lnrkvumxfbtwsejhdipcgqoa...
pjklwhntsgoqcdmvxairfube...

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